CHAPTER 4

Full-Wave Rectifiers

Converting ac to dc

4.1 INTRODUCTION

The objective of a full-wave rectifier is to produce a voltage or current that is purely dc or has some specified dc component. While the purpose of the full-wave rectifier is basically the same as that of the half-wave rectifier, full-wave rectifiers have some fundamental advantages.

The average current in the ac source is zero in the full-wave rectifier, thus avoiding problems associated with nonzero average source currents, particularly in transformers. The output of the full-wave rectifier has inherently less ripple than the half-wave rectifier.

4.2 SINGLE-PHASE FULL-WAVE RECTIFIERS

The bridge rectifier and the center-tapped transformer rectifier of Figs. 4-1 and 4-2 are two basic single-phase full-wave rectifiers.

The Bridge Rectifier

For the bridge rectifier of Fig. 4-1, these are some basic observations:

1. Diodes D_1 and D_2 conduct together, and D_3 and D_4 conduct together. Kirchhoff's voltage law around the loop containing the source, D_1, and D_3

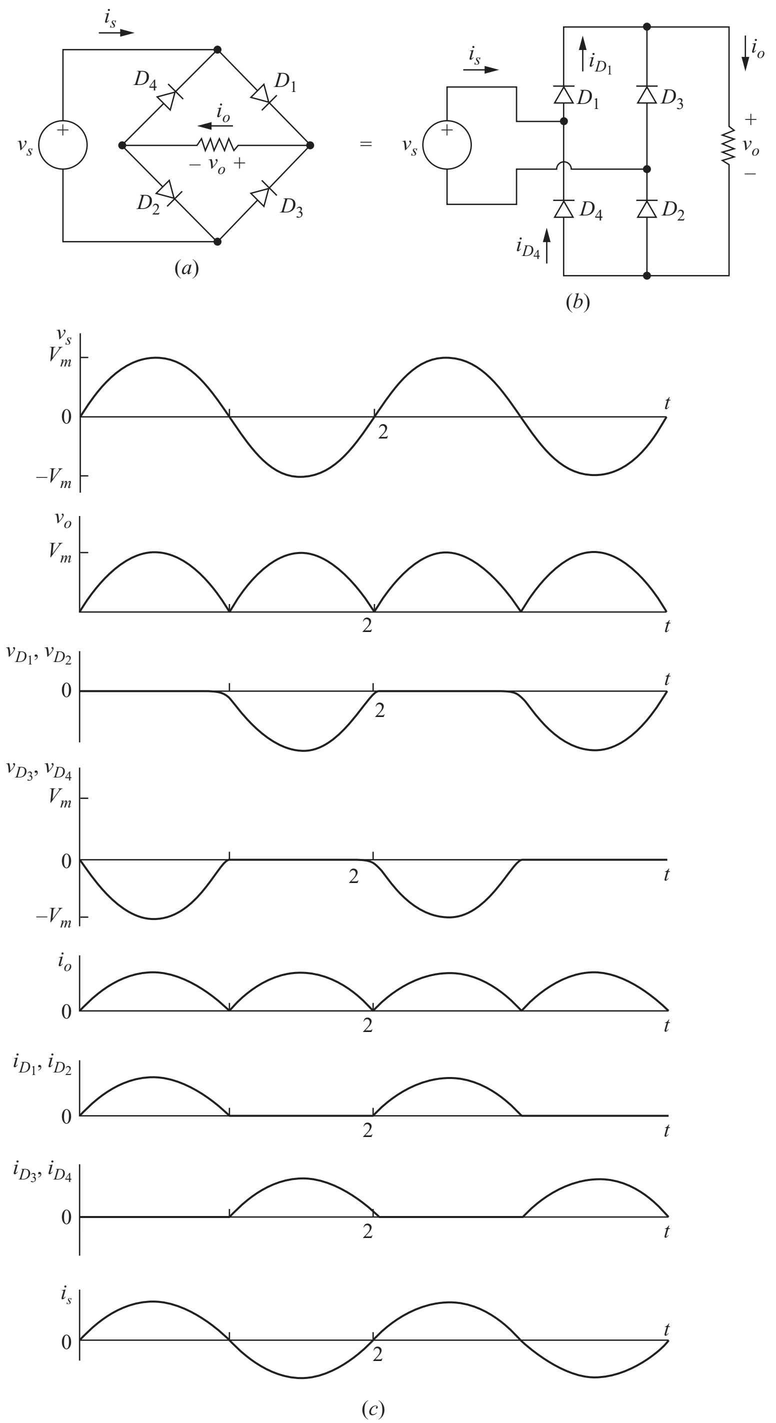

Figure 4-1 Full-wave bridge rectifier. (*a*) Circuit diagram. (*b*) Alternative representation. (*c*) Voltages and currents.

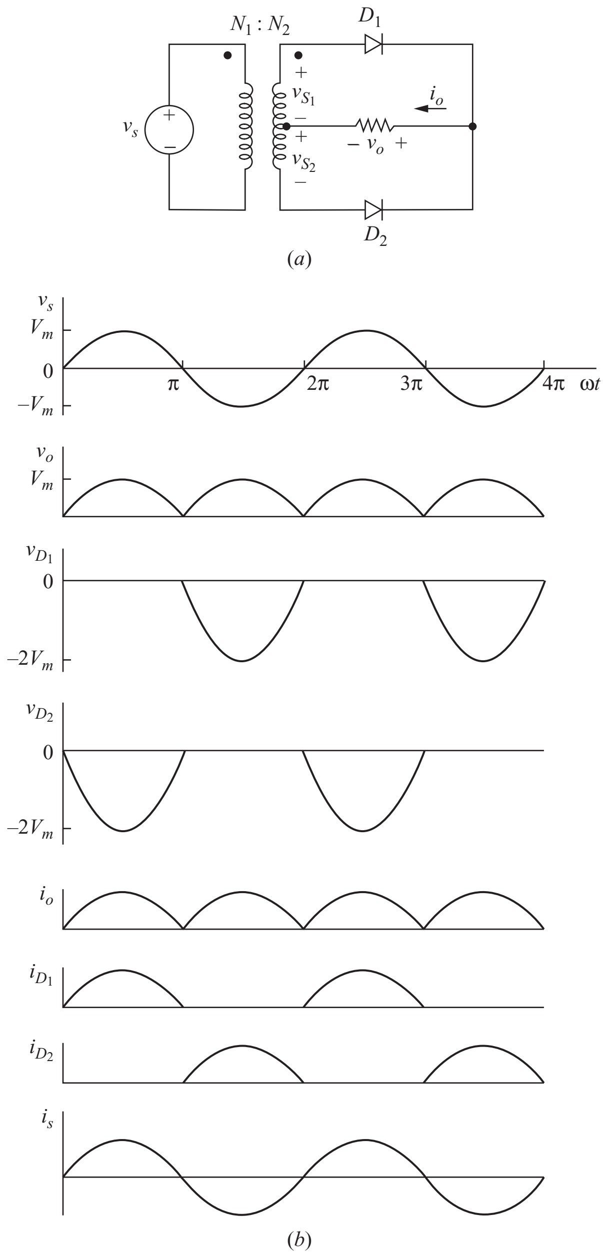

Figure 4-2 Full-wave center-tapped rectifier (*a*) circuit; (*b*) voltages and currents.

shows that D_1 and D_3 cannot be on at the same time. Similarly, D_2 and D_4 cannot conduct simultaneously. The load current can be positive or zero but can never be negative.

2. The voltage across the load is $+v_s$ when D_1 and D_2 are on. The voltage across the load is $-v_s$ when D_3 and D_4 are on.
3. The maximum voltage across a reverse-biased diode is the peak value of the source. This can be shown by Kirchhoff's voltage law around the loop containing the source, D_1, and D_3. With D_1 on, the voltage across D_3 is $-v_s$.
4. The current entering the bridge from the source is $i_{D_1} - i_{D_4}$, which is symmetric about zero. Therefore, the average source current is zero.
5. The rms source current is the same as the rms load current. The source current is the same as the load current for one-half of the source period and is the negative of the load current for the other half. The squares of the load and source currents are the same, so the rms currents are equal.
6. The fundamental frequency of the output voltage is 2ω, where ω is the frequency of the ac input since two periods of the output occur for every period of the input. The Fourier series of the output consists of a dc term and the even harmonics of the source frequency.

The Center-Tapped Transformer Rectifier

The voltage waveforms for a resistive load for the rectifier using the center-tapped transformer are shown in Fig. 4-2. Some basic observations for this circuit are as follows:

1. Kirchhoff's voltage law shows that only one diode can conduct at a time. Load current can be positive or zero but never negative.
2. The output voltage is $+v_{s_1}$ when D_1 conducts and is $-v_{s_2}$ when D_2 conducts. The transformer secondary voltages are related to the source voltage by $v_{s_1} = v_{s_2} = v_s(N_2/2N_1)$.
3. Kirchhoff's voltage law around the transformer secondary windings, D_1, and D_2 shows that the maximum voltage across a reverse-biased diode is *twice* the peak value of the load voltage.
4. Current in each half of the transformer secondary is reflected to the primary, resulting in an average source current of zero.
5. The transformer provides electrical isolation between the source and the load.
6. The fundamental frequency of the output voltage is 2ω since two periods of the output occur for every period of the input.

The lower peak diode voltage in the bridge rectifier makes it more suitable for high-voltage applications. The center-tapped transformer rectifier, in addition to including electrical isolation, has only one diode voltage drop between the source and load, making it desirable for low-voltage, high-current applications.

The following discussion focuses on the full-wave bridge rectifier but generally applies to the center-tapped circuit as well.

Resistive Load

The voltage across a resistive load for the bridge rectifier of Fig. 4-1 is expressed as

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } 0 \le \omega t \le \pi \\ -V_m \sin \omega t & \text{for } \pi \le \omega t \le 2\pi \end{cases} \tag{4-1}$$

The dc component of the output voltage is the average value, and load current is simply the resistor voltage divided by resistance.

$$\boxed{\begin{gathered} V_o = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t) = \frac{2V_m}{\pi} \\ I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R} \end{gathered}} \tag{4-2}$$

Power absorbed by the load resistor can be determined from $I^2_{\text{rms}}R$, where I_{rms} for the full-wave rectified current waveform is the same as for an unrectified sine wave,

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \tag{4-3}$$

The source current for the full-wave rectifier with a resistive load is a sinusoid that is in phase with the voltage, so the power factor is 1.

RL Load

For an *RL* series-connected load (Fig. 4-3*a*), the method of analysis is similar to that for the half-wave rectifier with the freewheeling diode discussed in Chap. 3. After a transient that occurs during start-up, the load current i_o reaches a periodic steady-state condition similar to that in Fig. 4-3*b*.

For the bridge circuit, current is transferred from one pair of diodes to the other pair when the source changes polarity. The voltage across the *RL* load is a full-wave rectified sinusoid, as it was for the resistive load. The full-wave rectified sinusoidal voltage across the load can be expressed as a Fourier series consisting of a dc term and the even harmonics

$$v_o(t) = V_o + \sum_{n=2,4\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

where (4-4)

$$V_o = \frac{2V_m}{\pi} \quad \text{and} \quad V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

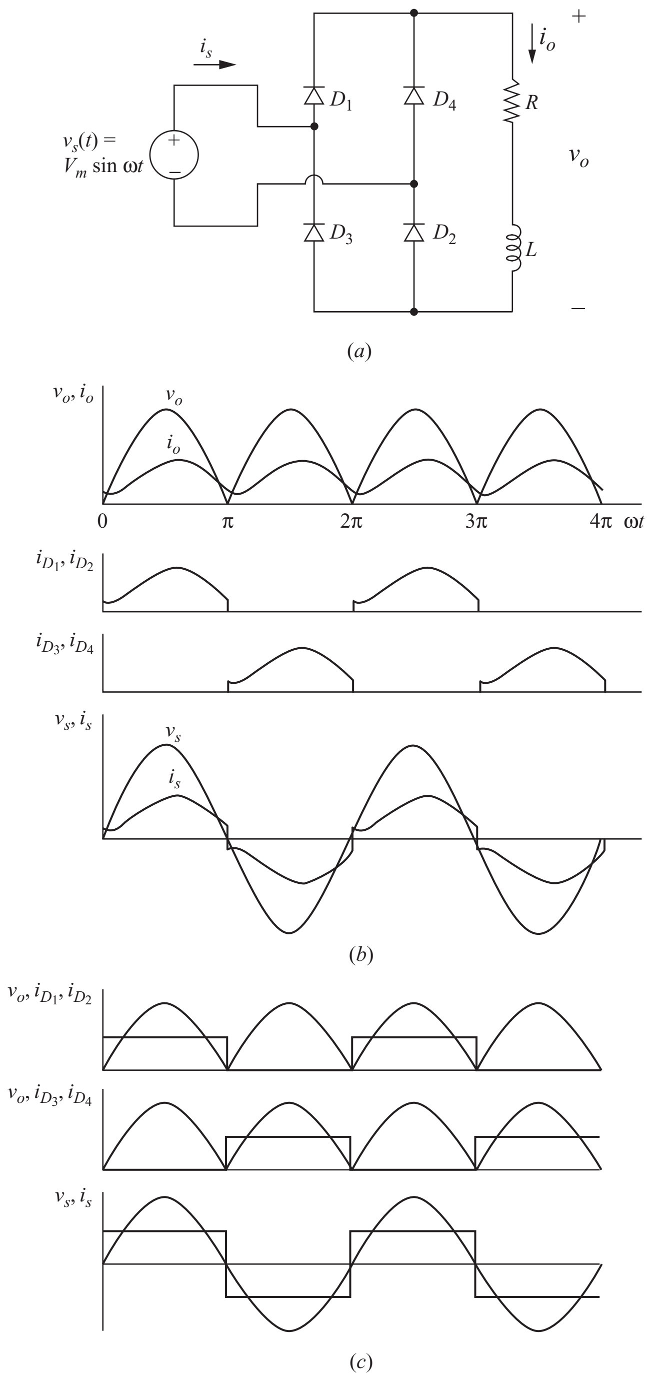

Figure 4-3 (*a*) Bridge rectifier with an *RL* load; (*b*) Voltages and currents; (*c*) Diode and source currents when the inductance is large and the current is nearly constant.

The current in the *RL* load is then computed using superposition, taking each frequency separately and combining the results. The dc current and current amplitude at each frequency are computed from

$$\boxed{\begin{aligned} I_0 &= \frac{V_0}{R} \\ I_n &= \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega L|} \end{aligned}} \tag{4-5}$$

Note that as the harmonic number n increases in Eq.(4-4), the voltage amplitude decreases. For an *RL* load, the impedance Z_n increases as n increases. The combination of decreasing V_n and increasing Z_n makes I_n decrease rapidly for increasing harmonic number. Therefore, the dc term and only a few, if any, of the ac terms are usually necessary to describe current in an *RL* load.

EXAMPLE 4-1

Full-Wave Rectifier with *RL* Load

The bridge rectifier circuit of Fig. 4-3*a* has an ac source with $V_m = 100$ V at 60 Hz and a series *RL* load with $R = 10\ \Omega$ and $L = 10$ mH. (*a*) Determine the average current in the load. (*b*) Estimate the peak-to-peak variation in load current based on the first ac term in the Fourier series. (*c*) Determine the power absorbed by the load and the power factor of the circuit. (*d*) Determine the average and rms currents in the diodes.

■ Solution

(*a*) The average load current is determined from the dc term in the Fourier series. The voltage across the load is a full-wave rectified sine wave that has the Fourier series determined from Eq. (4-4). Average output voltage is

$$V_0 = \frac{2V_m}{\pi} = \frac{2(200)}{\pi} = 63.7 \text{ V}$$

and average load current is

$$I_0 = \frac{V_0}{R} = \frac{63.7 \text{ V}}{10\ \Omega} = 6.37 \text{ A}$$

(*b*) Amplitudes of the ac voltage terms are determined from Eq. (4-4). For $n = 2$ and 4,

$$V_2 = \frac{2(100)}{\pi}\left(\frac{1}{1} - \frac{1}{3}\right) = 42.4 \text{ V}$$

$$V_4 = \frac{2(100)}{\pi}\left(\frac{1}{3} - \frac{1}{5}\right) = 8.49 \text{ V}$$

The amplitudes of first two ac current terms in the current Fourier series are computed from Eq. (4-5).

$$I_2 = \frac{42.4}{|10 + j(2)(377)(0.01)|} = \frac{42.4 \text{ V}}{12.5\ \Omega} = 3.39 \text{ A}$$

$$I_4 = \frac{8.49}{|10 + j(4)(377)(0.01)|} = \frac{8.49 \text{ V}}{18.1\ \Omega} = 0.47 \text{ A}$$

The current I_2 is much larger than I_4 and higher-order harmonics, so I_2 can be used to estimate the peak-to-peak variation in load current $\Delta i_o \approx 2(3.39) = 6.78$ A. Actual variation in i_o will be larger because of the higher-order terms.

(*c*) The power absorbed by the load is determined from I^2_{rms}. The rms current is then determined from Eq. (2-43) as

$$I_{\text{rms}} = \sqrt{\sum I^2_{n,\text{rms}}}$$

$$= \sqrt{(6.37)^2 + \left(\frac{3.39}{\sqrt{2}}\right)^2 + \left(\frac{0.47}{\sqrt{2}}\right)^2 + \cdots} \approx 6.81 \text{ A}$$

Adding more terms in the series would not be useful because they are small and have little effect on the result. Power in the load is

$$P = I^2_{\text{rms}} R = (6.81)^2(10) = 464 \text{ W}$$

The rms source current is the same as the rms load current. Power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{464}{\left(\frac{100}{\sqrt{2}}\right)(6.81)} = 0.964$$

(*d*) Each diode conducts for one-half of the time, so

$$I_{D,\text{avg}} = \frac{I_o}{2} = \frac{6.37}{2} = 3.19 \text{ A}$$

and

$$I_{D,\text{rms}} = \frac{I_{\text{rms}}}{\sqrt{2}} = \frac{6.81}{\sqrt{2}} = 4.82 \text{ A}$$

In some applications, the load inductance may be relatively large or made large by adding external inductance. If the inductive impedance for the ac terms in the Fourier series effectively eliminates the ac current terms in the load, the load current is essentially dc. If $\omega L \gg R$,

$$i(\omega t) \approx I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R} \qquad \text{for } \omega L \gg R \tag{4-6}$$

$$I_{\text{rms}} \approx I_o$$

Load and source voltages and currents are shown in Fig. 4-3*c*.

RL-Source Load

Another general industrial load may be modeled as a series resistance, inductance, and a dc voltage source, as shown in Fig. 4-5*a*. A dc motor drive circuit and a battery charger are applications for this model. There are two possible modes of operation for this circuit, the continuous-current mode and the discontinuous-current mode. In the continuous-current mode, the load current is always positive for steady-state operation (Fig. 4-5*b*). Discontinuous load current is characterized by current returning to zero during every period (Fig. 4-5*c*).

For continuous-current operation, one pair of diodes is always conducting, and the voltage across the load is a full-wave rectified sine wave. The only modification to the analysis that was done for an *RL* load is in the dc term of the Fourier series. The dc (average) component of current in this circuit is

$$I_o = \frac{V_o - V_{dc}}{R} = \frac{\dfrac{2V_m}{\pi} - V_{dc}}{R} \tag{4-7}$$

The sinusoidal terms in the Fourier analysis are unchanged by the dc source provided that the current is continuous.

Discontinuous current is analyzed like the half-wave rectifier of Sec. 3.5. The load voltage is not a full-wave rectified sine wave for this case, so the Fourier series of Eq. (4-4) does not apply.

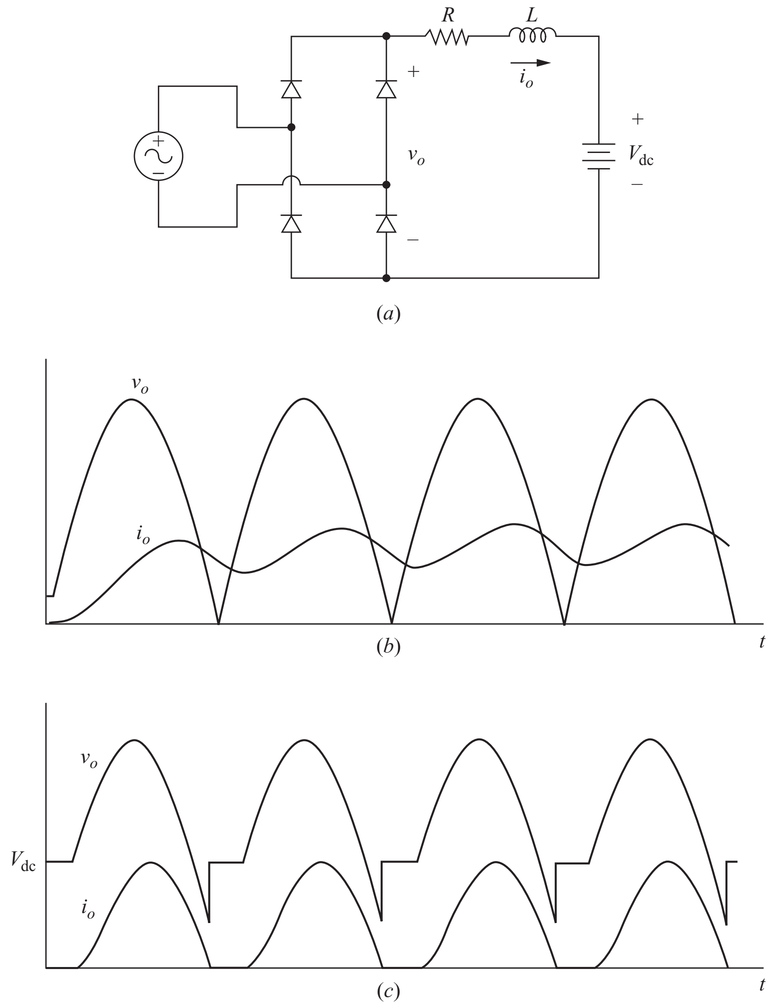

Figure 4-5 (*a*) Rectifier with *RL*-source load; (*b*) Continuous current: when the circuit is energized, the load current reaches the steady-state after a few periods; (*c*) Discontinuous current: the load current returns to zero during every period.

EXAMPLE 4-3

Full-Wave Rectifier with *RL*-Source Load—Continuous Current

For the full-wave bridge rectifier circuit of Fig. 4-5*a*, the ac source is 120 V rms at 60 Hz, $R = 2\ \Omega$, $L = 10$ mH, and $V_{dc} = 80$ V. Determine the power absorbed by the dc voltage source and the power absorbed by the load resistor.

■ Solution

For continuous current, the voltage across the load is a full-wave rectified sine wave which has the Fourier series given by Eq. (4-4). Equation (4-7) is used to compute the average current, which is used to compute power absorbed by the dc source,

$$I_0 = \frac{\dfrac{2V_m}{\pi} - V_{dc}}{R} = \frac{\dfrac{2\sqrt{2}(120)}{\pi} - 80}{2} = 14.0 \text{ A}$$

$$P_{dc} = I_0 V_{dc} = (14)(80) = 1120 \text{ W}$$

The first few terms of the Fourier series using Eqs. (4-4) and (4-5) are shown in Table 4-1.

Table 4-1 Fourier series components

n	V_n	Z_n	I_n
0	108	2.0	14.0
2	72.0	7.80	9.23
4	14.4	15.2	0.90

The rms current is computed from Eq. (2-43).

$$I_{rms} = \sqrt{14^2 + \left(\frac{9.23}{\sqrt{2}}\right)^2 + \left(\frac{0.90}{\sqrt{2}}\right)^2 + \cdots} \approx 15.46 \text{ A}$$

Power absorbed by the resistor is

$$P_R = I_{rms}^2 R = (15.46)^2(2) = 478 \text{ W}$$

Capacitance Output Filter

Placing a large capacitor in parallel with a resistive load can produce an output voltage that is essentially dc (Fig. 4-6). The analysis is very much like that of the half-wave rectifier with a capacitance filter in Chap. 3. In the full-wave circuit, the time that the capacitor discharges is smaller than that for the half-wave circuit

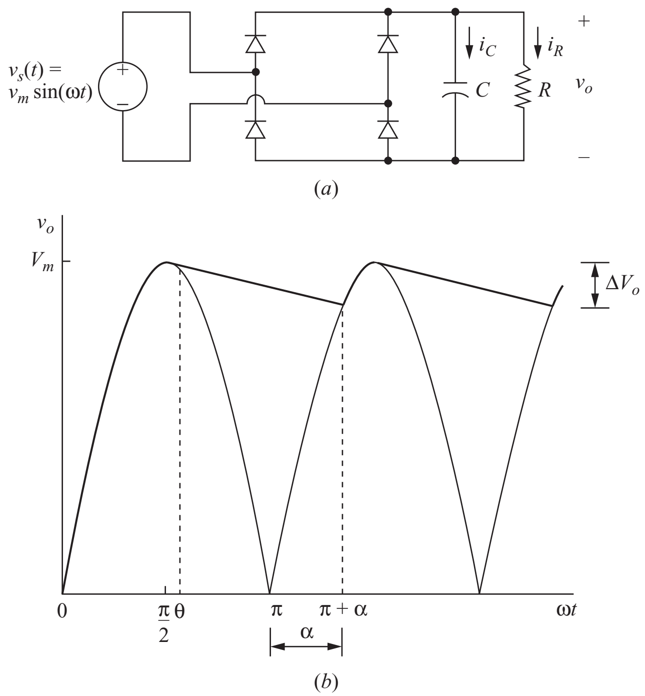

Figure 4-6 (*a*) Full-wave rectifier with capacitance filter; (*b*) Source and output voltage.

because of the rectified sine wave in the second half of each period. The output voltage ripple for the full-wave rectifier is approximately one-half that of the half-wave rectifier. The peak output voltage will be less in the full-wave circuit because there are two diode voltage drops rather than one.

The analysis proceeds exactly as for the half-wave rectifier. The output voltage is a positive sine function when one of the diode pairs is conducting and is a decaying exponential otherwise. Assuming ideal diodes,

$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin \theta)e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases} \tag{4-8}$$

where θ is the angle where the diodes become reverse biased, which is the same as that for the half-wave rectifier and is found using Eq. (3-41).

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi \tag{4-9}$$

The maximum output voltage is V_m, and the minimum output voltage is determined by evaluating v_o at the angle at which the second pair of diodes turns on, which is at $\omega t = \pi + \alpha$. At that boundary point,

$$(V_m \sin \theta)e^{-(\pi + \alpha - \theta)/\omega RC} = -V_m \sin(\pi + \alpha)$$

or

$$(\sin \theta)e^{-(\pi+\alpha-\theta)/\omega RC} - \sin \alpha = 0 \tag{4-10}$$

which must be solved numerically for α.

The peak-to-peak voltage variation, or ripple, is the difference between maximum and minimum voltages.

$$\boxed{\Delta V_o = V_m - |V_m \sin(\pi + \alpha)| = V_m(1 - \sin \alpha)} \tag{4-11}$$

This is the same as Eq. (3-49) for voltage variation in the half-wave rectifier, but α is larger for the full-wave rectifier and the ripple is smaller for a given load. Capacitor current is described by the same equations as for the half-wave rectifier.

In practical circuits where $\omega RC \gg \pi$.

$$\theta \approx \pi/2 \qquad \alpha \approx \pi/2 \tag{4-12}$$

The minimum output voltage is then approximated from Eq. (4-9) for the diodes off evaluated at $\omega t = \pi$.

$$v_o(\pi + \alpha) = V_m e^{-(\pi+\pi/2-\pi/2)/\omega RC} = V_m e^{-\pi/\omega RC}$$

The ripple voltage for the full-wave rectifier with a capacitor filter can then be approximated as

$$\Delta V_o \approx V_m(1 - e^{-\pi/\omega RC})$$

Furthermore, the exponential in the above equation can be approximated by the series expansion

$$e^{-\pi/\omega RC} \approx 1 - \frac{\pi}{\omega RC}$$

Substituting for the exponential in the approximation, the peak-to-peak ripple is

$$\boxed{\Delta V_o \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}} \tag{4-13}$$

Note that the approximate peak-to-peak ripple voltage for the full-wave rectifier is one-half that of the half-wave rectifier from Eq. (3-51). As for the half-wave rectifier, the peak diode current is much larger than the average diode current and Eq. (3-48) applies. The average source current is zero.

EXAMPLE 4-4

Full-Wave Rectifier with Capacitance Filter

The full-wave rectifier of Fig. 4-6*a* has a 120 V source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu$F. (*a*) Determine the peak-to-peak voltage variation of the output. (*b*) Determine the value of capacitance that would reduce the output voltage ripple to 1percent of the dc value.

■ **Solution**

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85$$

The angle θ is determined from Eq. (4-9).

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62 \text{ rad} = 93°$$

$$V_m \sin\theta = 169.5 \text{ V}$$

The angle α is determined by the numerical solution of Eq. (4-10).

$$\sin(1.62)e^{-(\pi+\alpha-1.62)/18.85} - \sin\alpha = 0$$

$$\alpha = 1.06 \text{ rad} = 60.6°$$

(*a*) Peak-to-peak output voltage is described by Eq. (4-11).

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7[1 - \sin(1.06)] = 22 \text{ V}$$

Note that this is the same load and source as for the half-wave rectifier of Example 3-9 where $\Delta V_o = 43$ V.

(*b*) With the ripple limited to 1 percent, the output voltage will be held close to V_m and the approximation of Eq. (4-13) applies.

$$\frac{\Delta V_o}{V_m} = 0.01 \approx \frac{1}{2fRC}$$

Solving for *C*,

$$C \approx \frac{1}{2fR(\Delta V_o/V_m)} = \frac{1}{(2)(60)(500)(0.01)} = 1670\ \mu\text{F}$$

EXAMPLE 4-5

Full-Wave Rectifier with *LC* Filter

A full-wave rectifier has a source of $v_s(t) = 100 \sin(377t)$ V. An *LC* filter as in Fig. 4-8*a* is used, with $L = 5$ mH and $C = 10{,}000$ μF. The load resistance is (*a*) 5 Ω and (*b*) 50 Ω. Determine the output voltage for each case.

■ Solution

Using Eq. (4-17), continuous inductor current exists when

$$R < 3\omega L = 3(377)(0.005) = 5.7\ \Omega$$

which indicates continuous current for 5 Ω and discontinuous current for 50 Ω.

(*a*) For $R = 5\ \Omega$ with continuous current, output voltage is determined from Eq. (4-14).

$$V_o = \frac{2V_m}{\pi} = \frac{2(100)}{\pi} = 63.7\text{ V}$$

(*b*) For $R = 50\ \Omega$ with discontinuous current, the iteration method is used to determine V_o. Initially, V_o is estimated to be 90 V. The results of the iteration are as follows:

Estimated V_o	α	β	Calculated V_o	
90	1.12	2.48	38.8	(Estimate is too high)
80	0.93	2.89	159	(Estimate is too low)
85	1.12	2.70	88.2	(Estimate is slightly low)
86	1.04	2.66	76.6	(Estimate is too high)
85.3	1.02	2.69	84.6	(Approximate solution)

Therefore, V_o is approximately 85.3 V. As a practical matter, three significant figures for the load voltage may not be justified when predicting performance of a real circuit. Knowing that the output voltage is slightly above 85 V after the third iteration is probably sufficient. Output could also be estimated from the graph of Fig. 4-8*d*.