4.3 CONTROLLED FULL-WAVE RECTIFIERS

A versatile method of controlling the output of a full-wave rectifier is to substitute controlled switches such as thyristors (SCRs) for the diodes. Output is controlled by adjusting the delay angle of each SCR, resulting in an output voltage that is adjustable over a limited range.

Controlled full-wave rectifiers are shown in Fig. 4-10. For the bridge rectifier, SCRs S_1 and S_2 will become forward-biased when the source becomes positive but will not conduct until gate signals are applied. Similarly, S_3 and S_4 will become forward-biased when the source becomes negative but will not conduct until they receive gate signals. For the center-tapped transformer rectifier, S_1 is forward-biased when v_s is positive, and S_2 is forward-biased when v_s is negative, but each will not conduct until it receives a gate signal.

The delay angle α is the angle interval between the forward biasing of the SCR and the gate signal application. If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes. The discussion that follows generally applies to both bridge and center-tapped rectifiers.

Resistive Load

The output voltage waveform for a controlled full-wave rectifier with a resistive load is shown in Fig. 4-10*c*. The average component of this waveform is determined from

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}(1 + \cos\alpha)} \tag{4-23}$$

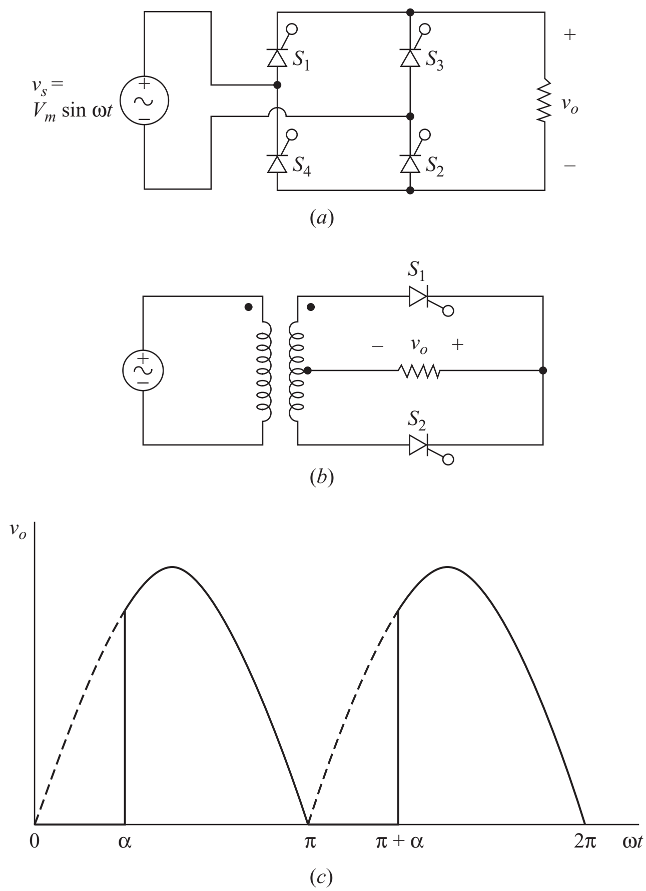

Figure 4-10 (*a*) Controlled full-wave bridge rectifier;
(*b*) Controlled full-wave center-tapped transformer rectifier;
(*c*) Output for a resistive load.

Average output current is then

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos \alpha) \tag{4-24}$$

The power delivered to the load is a function of the input voltage, the delay angle, and the load components; $P = I_{\text{rms}}^2 R$ is used to determine the power in a resistive load, where

$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin \omega t\right)^2 d(\omega t)} \\ &= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}} \end{aligned} \tag{4-25}$$

The rms current in the source is the same as the rms current in the load.

EXAMPLE 4-6

Controlled Full-Wave Rectifier with Resistive Load

The full-wave controlled bridge rectifier of Fig. 4-10*a* has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

■ Solution

The average output voltage is determined from Eq. (4-23).

$$V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40°\right) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current from Eq. (4-24), remembering to use α in radians.

$$I_{\text{rms}} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{\text{rms}}^2 R = (5.80)^2(20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

RL Load, Discontinuous Current

Load current for a controlled full-wave rectifier with an *RL* load (Fig. 4-11*a*) can be either continuous or discontinuous, and a separate analysis is required for each. Starting the analysis at $\omega t = 0$ with zero load current, SCRs S_1 and S_2 in the bridge rectifier will be forward-biased and S_3 and S_4 will be reverse-biased as the source voltage becomes positive. Gate signals are applied to S_1 and S_2 at $\omega t = \alpha$, turning S_1 and S_2 on. With S_1 and S_2 on, the load voltage is equal to the source voltage. For this condition, the circuit is identical to that of the controlled half-wave rectifier of Chap. 3, having a current function

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

where (4-26)

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

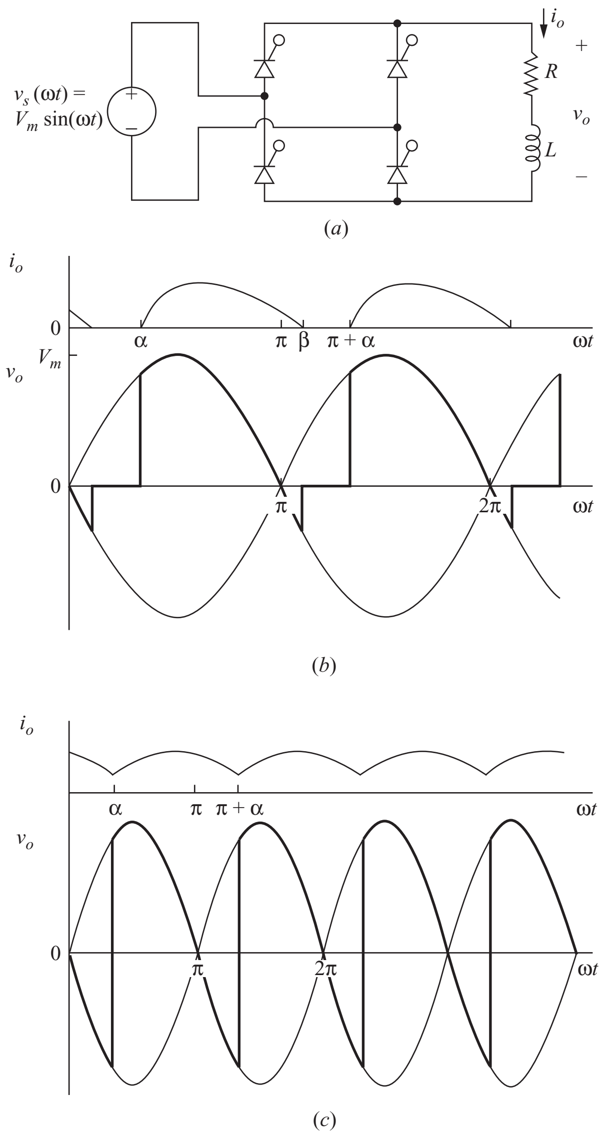

Figure 4-11 (*a*) Controlled rectifier with *RL* load; (*b*) Discontinuous current; (*c*) Continuous current.

The above current function becomes zero at $\omega t = \beta$. If $\beta < \pi + \alpha$, the current remains at zero until $\omega t = \pi + \alpha$ when gate signals are applied to S_3 and S_4 which are then forward-biased and begin to conduct. This mode of operation is called *discontinuous current*, which is illustrated in Fig. 4-11*b*.

$$\beta < \alpha + \pi \quad \rightarrow \quad \text{discontinuous current} \tag{4-27}$$

Analysis of the controlled full-wave rectifier operating in the discontinuous-current mode is identical to that of the controlled half-wave rectifier except that the period for the output current is π rather than 2π rad.

EXAMPLE 4-7

Controlled Full-Wave Rectifier, Discontinuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, $R = 10\ \Omega$, $L = 20$ mH, and $\alpha = 60°$. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

■ **Solution**

From the parameters given,

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

(*a*) Substituting into Eq. (4-26),

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \text{ A} \qquad \text{for } \alpha \le \omega t \le \beta$$

Solving $i_o(\beta) = 0$ numerically for β, $\beta = 3.78$ rad (216°). Since $\pi + \alpha = 4.19 > \beta$, the current is discontinuous, and the above expression for current is valid.

(*b*) Average load current is determined from the numerical integration of

$$I_o = \frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t) = 7.05 \text{ A}$$

(*c*) Power absorbed by the load occurs in the resistor and is computed from $I_{\text{rms}}^2 R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t)} = 8.35 \text{ A}$$

$$P = (8.35)^2(10) = 697 \text{ W}$$

RL Load, Continuous Current

If the load current is still positive at $\omega t = \pi + \alpha$ when gate signals are applied to S_3 and S_4 in the above analysis, S_3 and S_4 are turned on and S_1 and S_2 are forced off. Since the initial condition for current in the second half-cycle is not zero, the current function does not repeat. Equation (4-26) is not valid in the steady state for continuous current. For an *RL* load with continuous current, the steady-state current and voltage waveforms are generally as shown in Fig. 4-11*c*.

The boundary between continuous and discontinuous current occurs when β for Eq. (4-26) is $\pi + \alpha$. The current at $\omega t = \pi + \alpha$ must be greater than zero for continuous-current operation.

$$i(\pi + \alpha) \geq 0$$

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta)\, e^{-(\pi+\alpha-\alpha)/\omega\tau} \geq 0$$

Using

$$\sin(\pi + \alpha - \theta) = \sin(\theta - \alpha)$$

$$\sin(\theta - \alpha)\left(1 - e^{-(\pi/\omega\tau)}\right) \geq 0$$

Solving for α,

$$\alpha \leq \theta$$

Using

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{for continuous current} \tag{4-28}$$

Either Eq. (4-27) or Eq. (4-28) can be used to check whether the load current is continuous or discontinuous.

A method for determining the output voltage and current for the continuous-current case is to use the Fourier series. The Fourier series for the voltage waveform for continuous-current case shown in Fig. 4-11*c* is expressed in general form as

$$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n) \tag{4-29}$$

The dc (average) value is

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha} \tag{4-30}$$

The amplitudes of the ac terms are calculated from

$$V_n = \sqrt{a_n^2 + b_n^2} \tag{4-31}$$

where

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right] \tag{4-32}$$

$$n = 2, 4, 6, \ldots$$

Figure 4-12 shows the relationship between normalized harmonic content of the output voltage and delay angle.

The Fourier series for current is determined by superposition as was done for the uncontrolled rectifier earlier in this chapter. The current amplitude at each frequency is determined from Eq. (4-5). The rms current is determined by combining the rms currents at each frequency. From Eq. (2-43),

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2}$$

where

$$I_o = \frac{V_o}{R} \quad \text{and} \quad I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega_0 L|} \tag{4-33}$$

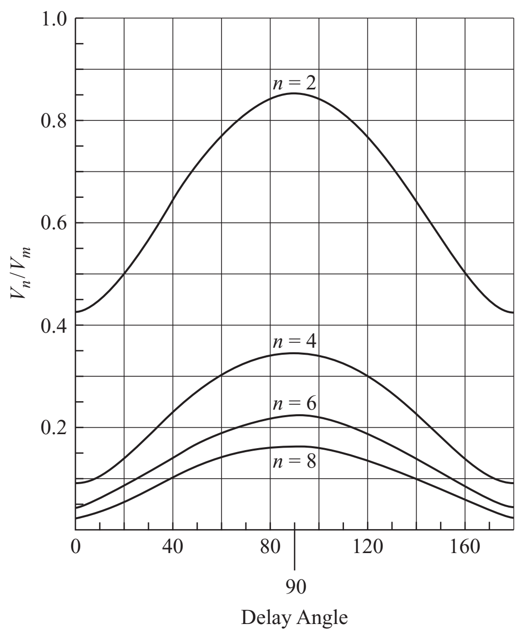

Figure 4-12 Output harmonic voltages as a function of delay angle for a single-phase controlled rectifier.

As the harmonic number increases, the impedance for the inductance increases. Therefore, it may be necessary to solve for only a few terms of the series to be able to calculate the rms current. If the inductor is large, the ac terms will become small, and the current is essentially dc.

EXAMPLE 4-8

Controlled Full-Wave Rectifier with *RL* Load, Continuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, an *RL* load where $R = 10\ \Omega$ and $L = 100$ mH. The delay angle $\alpha = 60°$ (same as Example 4-7 except L is larger). (*a*) Verify that the load current is continuous. (*b*) Determine the dc (average) component of the current. (*c*) Determine the power absorbed by the load.

■ Solution

(*a*) Equation (4-28) is used to verify that the current is continuous.

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75°$$

$$\alpha = 60° < 75° \qquad \therefore \text{ continuous current}$$

(*b*) The voltage across the load is expressed in terms of the Fourier series of Eq. (4-29). The dc term is computed from Eq. (4-30).

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}(120)}{\pi}\cos(60°) = 54.0 \text{ V}$$

(*c*) The amplitudes of the ac terms are computed from Eqs. (4-31) and (4-32) and are summarized in the following table where, $Z_n = |R + j\omega L|$ and $I_n = V_n/Z_n$.

n	a_n	b_n	V_n	Z_n	I_n
0 (dc)	—	—	54.0	10	5.40
2	−90	−93.5	129.8	76.0	1.71
4	46.8	−18.7	50.4	151.1	0.33
6	−3.19	32.0	32.2	226.4	0.14

The rms current is computed from Eq. (4-33).

$$I_{\text{rms}} = \sqrt{(5.40)^2 + \left(\frac{1.71}{\sqrt{2}}\right)^2 + \left(\frac{0.33}{\sqrt{2}}\right)^2 + \left(\frac{0.14}{\sqrt{2}}\right)^2 + \cdots} \approx 5.54 \text{ A}$$

Power is computed from $I_{\text{rms}}^2 R$.

$$P = (5.54)^2(10) = 307 \text{ W}$$

Note that the rms current could be approximated accurately from the dc term and one ac term ($n = 2$). Higher-frequency terms are very small and contribute little to the power in the load.

Controlled Rectifier with *RL*-Source Load

The controlled rectifier with a load that is a series resistance, inductance, and dc voltage (Fig. 4-14) is analyzed much like the uncontrolled rectifier of Fig. 4-5*a* discussed earlier in this chapter. For the controlled rectifier, the SCRs may be turned on at any time that they are forward-biased, which is at an angle

$$\alpha \geq \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) \tag{4-34}$$

For the continuous-current case, the bridge output voltage is the same as in Fig. 4-11*c*. The average bridge output voltage is

$$V_o = \frac{2V_m}{\pi}\cos\alpha \tag{4-35}$$

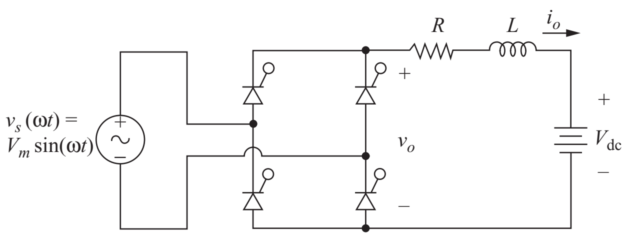

Figure 4-14 Controlled rectifier with *RL*-source load.

The average load current is

$$I_o = \frac{V_o - V_{dc}}{R} \tag{4-36}$$

The ac voltage terms are unchanged from the controlled rectifier with an *RL* load in Fig. 4-11*a* and are described by Eqs. (4-29) to (4-32). The ac current terms are determined from the circuit of Fig. 4-14*c*. Power absorbed by the dc voltage is

$$P_{dc} = I_o V_{dc} \tag{4-37}$$

Power absorbed by the resistor in the load is $I_{rms}^2 R$. If the inductance is large and the load current has little ripple, power absorbed by the resistor is approximately $I_o^2 R$.

EXAMPLE 4-10

Controlled Rectifier with *RL*-Source Load

The controlled rectifier of Fig. 4-14 has an ac source of 240 V rms at 60 Hz, $V_{dc} = 100$ V, $R = 5\ \Omega$, and an inductor large enough to cause continuous current. (*a*) Determine the delay angle α such that the power absorbed by the dc source is 1000 W. (*b*) Determine the value of inductance that will limit the peak-to-peak load current variation to 2 A.

■ Solution

(*a*) For the power in the 100-V dc source to be 1000 W, the current in it must be 10 A. The required output voltage is determined from Eq. (4-36) as

$$V_o = V_{dc} + I_o R = 100 + (10)(5) = 150 \text{ V}$$

The delay angle which will produce a 150 V dc output from the rectifier is determined from Eq. (4-35).

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{(150)(\pi)}{2\sqrt{2}(240)}\right] = 46°$$

(*b*) Variation in load current is due to the ac terms in the Fourier series. The load current amplitude for each of the ac terms is

$$I_n = \frac{V_n}{Z_n}$$

where V_n is described by Eqs. (4-31) and (4-32) or can be estimated from the graph of Fig. 4-12. The impedance for the ac terms is

$$Z_n = |R + jn\omega_0 L|$$

Since the decreasing amplitude of the voltage terms and the increasing magnitude of the impedance both contribute to diminishing ac currents as n increases, the peak-to-peak current variation will be estimated from the first ac term. For $n = 2$, V_n/V_m is estimated from Fig. 4-12 as 0.68 for $\alpha = 46°$, making $V_2 = 0.68V_m = 0.68\ (240\sqrt{2}) = 230$ V. The peak-to-peak variation of 2 A corresponds to a 1-A zero-to-peak amplitude. The required load impedance for $n = 2$ is then

$$Z_2 = \frac{V_2}{I_2} = \frac{230 \text{ V}}{1 \text{ A}} = 230\ \Omega$$

The 5-Ω resistor is insignificant compared to the total 230-Ω required impedance, so $Z_n \approx n\omega L$. Solving for L,

$$L \approx \frac{Z_2}{2\omega} = \frac{230}{2(377)} = 0.31 \text{ H}$$

A slightly larger inductance should be chosen to allow for the effect of higher-order ac terms.

Controlled Single-Phase Converter Operating as an Inverter

The above discussion focused on circuits operating as rectifiers, which means that the power flow is from the ac source to the load. It is also possible for power to flow from the load to the ac source, which classifies the circuit as an inverter.

For inverter operation of the converter in Fig. 4-14, power is supplied by the dc source, and power is absorbed by the bridge and is transferred to the ac system. The load current must be in the direction shown because of the SCRs in the bridge. For power to be supplied by the dc source, V_{dc} must be negative. For power to be absorbed by the bridge and transferred to the ac system, the bridge output voltage V_o must also be negative. Equation (4-35) applies, so a delay angle larger than 90° will result in a negative output voltage.

$$\begin{aligned} 0 < \alpha < 90° \rightarrow V_o > 0 \qquad &\text{rectifier operation} \\ 90° < \alpha < 180° \rightarrow V_o < 0 \qquad &\text{inverter operation} \end{aligned} \tag{4-38}$$

The voltage waveform for $\alpha = 150°$ and continuous inductor current is shown in Fig. 4-15. Equations (4-36) to (4-38) apply. If the inductor is large enough to

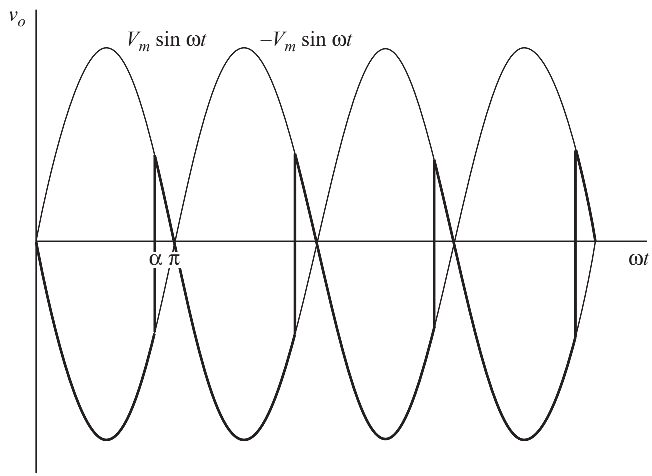

Figure 4-15 Output voltage for the controlled single-phase converter of Fig. 4-14 operating as an inverter, $\alpha = 150°$ and $V_{dc} < 0$.

effectively eliminate the ac current terms and the bridge is lossless, the power absorbed by the bridge and transferred to the ac system is

$$P_{bridge} = P_{ac} = -I_o V_o \tag{4-39}$$

EXAMPLE 4-11

Single-Phase Bridge Operating as an Inverter

The dc voltage in Fig. 4-14 represents the voltage generated by an array of solar cells and has a value of 110 V, connected such that $V_{dc} = -110$ V. The solar cells are capable of producing 1000 W. The ac source is 120 V rms, $R = 0.5\ \Omega$, and L is large enough to cause the load current to be essentially dc. Determine the delay angle α such that 1000 W is supplied by the solar cell array. Determine the power transferred to the ac system and the losses in the resistance. Assume ideal SCRs.

■ Solution

For the solar cell array to supply 1000 W, the average current must be

$$I_o = \frac{P_{dc}}{V_{dc}} = \frac{1000}{110} = 9.09 \text{ A}$$

The average output voltage of the bridge is determined from Eq. (4-36).

$$V_o = I_o R + V_{dc} = (9.09)(0.5) + (-110) = -105.5 \text{ V}$$

The required delay angle is determined from Eq. (4-35).

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{-105.5\pi}{2\sqrt{2}(120)}\right] = 165.5°$$

Power absorbed by the bridge and transferred to the ac system is determined from Eq. (4-39).

$$P_{ac} = -V_o I_o = (-9.09)(-105.5) = 959 \text{ W}$$

Power absorbed by the resistor is

$$P_R = I_{rms}^2 R \approx I_o^2 R = (9.09)^2(0.5) = 41 \text{ W}$$

Note that the load current and power will be sensitive to the delay angle and the voltage drops across the SCRs because bridge output voltage is close to the dc source voltage. For example, assume that the voltage across a conducting SCR is 1 V. Two SCRs conduct at all times, so the average bridge output voltage is reduced to

$$V_o = -105.5 - 2 = -107.5 \text{ V}$$

Average load current is then

$$I_o = \frac{-107.5 - (-110)}{0.5} = 5.0 \text{ A}$$

Power delivered to the bridge is then reduced to

$$P_{\text{bridge}} = (107.5)(5.0) = 537.5 \text{ W}$$

Average current in each SCR is one-half the average load current. Power absorbed by each SCR is approximately

$$P_{\text{SCR}} = I_{\text{SCR}} V_{\text{SCR}} = \frac{1}{2} I_o V_{\text{SCR}} = \frac{1}{2}(5)(1) = 2.5 \text{ W}$$

Total power loss in the bridge is then $4(2.5) = 10$ W, and power delivered to the ac source is $537.5 - 10 = 527.5$ W.

Problems

Uncontrolled Single-Phase Rectifiers

4-1. A single-phase full-wave bridge rectifier has a resistive load of 18 Ω and an ac source of 120-V rms. Determine the average, peak, and rms currents in the load and in each diode.

4-2. A single-phase rectifier has a resistive load of 25 Ω. Determine the average current and peak reverse voltage across each of the diodes for (*a*) a bridge rectifier with an ac source of 120 V rms and 60 Hz and (*b*) a center-tapped transformer rectifier with 120 V rms on each half of the secondary winding.

4-3. A single-phase bridge rectifier has an RL load with $R = 15\ \Omega$ and $L = 60$ mH. The ac source is $v_s = 100 \sin(377t)$ V. Determine the average and rms currents in the load and in each diode.

4-4. A single-phase bridge rectifier has an RL load with $R = 10\ \Omega$ and $L = 25$ mH. The ac source is $v_s = 170 \sin(377t)$ V. Determine the average and rms currents in the load and in each diode.

4-5. A single-phase bridge rectifier has an RL load with $R = 15\ \Omega$ and $L = 30$ mH. The ac source is 120 V rms, 60 Hz. Determine (*a*) the average load current, (*b*) the power absorbed by the load, and (*c*) the power factor.

4-6. A single-phase bridge rectifier has an RL load with $R = 12\ \Omega$ and $L = 20$ mH. The ac source is 120 V rms and 60 Hz. Determine (*a*) the average load current, (*b*) the power absorbed by the load, and (*c*) the power factor.

4-7. A single-phase center-tapped transformer rectifier has an ac source of 240 V rms and 60 Hz. The overall transformer turns ratio is 3:1 (80 V between the extreme ends of the secondary and 40 V on each tap). The load is a resistance of 4 Ω. Determine (*a*) the average load current, (*b*) the rms load current, (*c*) the average source current, and (*d*) the rms source current. Sketch the current waveforms of the load and the source.

4-8. Design a center-tapped transformer rectifier to produce an average current of 10.0 A in a 15-Ω resistive load. Both 120- and 240-V rms 60-Hz sources are available. Specify which source to use and specify the turns ratio of the transformer.

4-9. Design a center-tapped transformer rectifier to produce an average current of 5.0 A in an RL load with $R = 10\ \Omega$ and $L = 50$ mH. Both 120- and 240-V rms 60-Hz sources are available. Specify which source to use and specify the turns ratio of the transformer.

4-10. An electromagnet is modeled as a 200-mH inductance in series with a 4-Ω resistance. The average current in the inductance must be 10 A to establish the required magnetic field. Determine the amount of additional series resistance required to produce the required average current from a bridge rectifier supplied from a single-phase 120-V, 60-Hz source.

4-11. The full-wave rectifier of Fig. 4-3*a* has $v_s(\omega t) = 170 \sin \omega t$ V, $R = 3\ \Omega$, $L = 15$ mH, $V_{dc} = 48$ V, and $\omega = 2\pi(60)$ rad/s. Determine (*a*) the power absorbed by the dc source, (*b*) the power absorbed by the resistor, and (*c*) the power factor. (*d*) Estimate the peak-to-peak variation in the load current by considering only the first ac term in the Fourier series for current.

4-12. The full-wave rectifier of Fig. 4-3*a* has $v_s(\omega t) = 340 \sin \omega t$ V, $R = 5\ \Omega$, $L = 40$ mH, $V_{dc} = 96$ V, and $\omega = 2\pi(60)$ rad/s. Determine (*a*) the power absorbed by the dc source, (*b*) the power absorbed by the resistor, and (*c*) the power factor. (*d*) Estimate the peak-to-peak variation in the load current by considering only the first ac term in the Fourier series for current.

4-15. The single-phase full-wave bridge rectifier of Fig. 4-5*a* has an RL-source load with $R = 4\ \Omega$, $L = 40$ mH, and $V_{dc} = 24$ V. The ac source is 120 V rms at 60 Hz. Determine (*a*) the power absorbed by the dc source, (*b*) the power absorbed by the resistor, and (*c*) the power factor.

4-16. The single-phase full-wave bridge rectifier of Fig. 4-5*a* has an *RL*-source load with $R = 5\ \Omega$, $L = 60$ mH, and $V_{dc} = 36$ V. The ac source is 120 V rms at 60 Hz. Determine (*a*) the power absorbed by the dc source, (*b*) the power absorbed by the resistor, and (*c*) the power factor.

4-18. The full-wave rectifier of Fig. 4-6 has a 120-V rms 60 Hz source and a load resistance of 200 Ω. Determine the filter capacitance required to limit the peak-to-peak output voltage ripple to 1 percent of the dc output. Determine the peak and average diode currents.

4-19. The full-wave rectifier of Fig. 4-6 has a 60-Hz ac source with $V_m = 100$ V. It is to supply a load that requires a dc voltage of 100 V and will draw 0.5 A. Determine the filter capacitance required to limit the peak-to-peak output voltage ripple to 1 percent of the dc output. Determine the peak and average diode currents.

4-20. In Example 3-9, the half-wave rectifier of Fig. 3-11*a* has a 120 V rms source at 60 Hz, $R = 500\ \Omega$. The capacitance required for a 1 percent ripple in output voltage was determined to be 3333 μF. Determine the capacitance required for a 1 percent ripple if a full-wave rectifier is used instead. Determine the peak diode currents for each circuit. Discuss the advantages and disadvantages of each circuit.

Controlled Single-phase Rectifiers

4-23. The controlled single-phase bridge rectifier of Fig. 4-10*a* has a 20-Ω resistive load and has a 120-V rms, 60-Hz ac source. The delay angle is 45°. Determine (*a*) the average load current, (*b*) the rms load current, (*c*) the rms source current, and (*d*) the power factor.

4-24. Show that the power factor for the controlled full-wave rectifier with a resistive load is

$$\text{pf} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

4-25. The controlled single-phase full-wave bridge rectifier of Fig. 4-11*a* has an *RL* load with $R = 25\ \Omega$ and $L = 50$ mH. The source is 240 V rms at 60 Hz. Determine the average load current for (*a*) $\alpha = 15°$ and (*b*) $\alpha = 75°$.

4-26. The controlled single-phase full-wave bridge rectifier of Fig. 4-11*a* has an *RL* load with $R = 30\ \Omega$ and $L = 75$ mH. The source is 120 V rms at 60 Hz. Determine the average load current for (*a*) $\alpha = 20°$ and (*b*) $\alpha = 80°$.