CHAPTER 5

AC Voltage Controllers

AC to ac Converters

5.1 INTRODUCTION

An ac voltage controller is a converter that controls the voltage, current, and average power delivered to an ac load from an ac source. Electronic switches connect and disconnect the source and the load at regular intervals. In a switching scheme called phase control, switching takes place during every cycle of the source, in effect removing some of the source waveform before it reaches the load. Another type of control is integral-cycle control, whereby the source is connected and disconnected for several cycles at a time.

The phase-controlled ac voltage controller has several practical uses including light-dimmer circuits and speed control of induction motors. The input voltage source is ac, and the output is ac (although not sinusoidal), so the circuit is classified as an ac-ac converter.

5.2 THE SINGLE-PHASE AC VOLTAGE CONTROLLER

Basic Operation

A basic single-phase voltage controller is shown in Fig. 5-1*a*. The electronic switches are shown as parallel thyristors (SCRs). This SCR arrangement makes it possible to have current in either direction in the load. This SCR connection is called antiparallel or inverse parallel because the SCRs carry current in opposite directions. A triac is equivalent to the antiparallel SCRs. Other controlled switching devices can be used instead of SCRs.

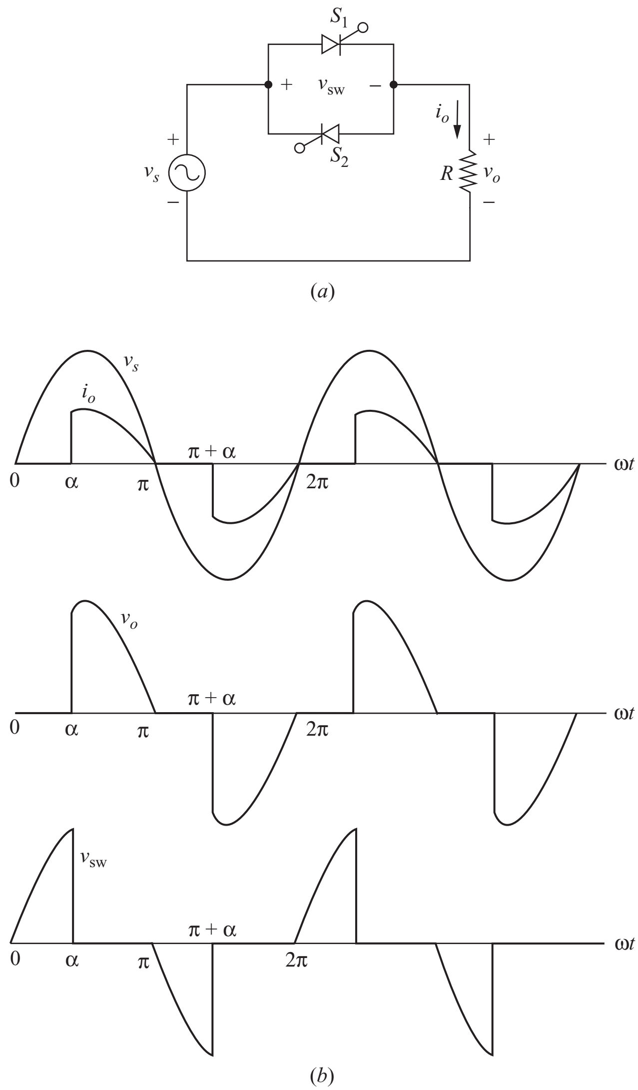

Figure 5-1 (*a*) Single-phase ac voltage controller with a resistive load; (*b*) Waveforms.

The principle of operation for a single-phase ac voltage controller using phase control is quite similar to that of the controlled half-wave rectifier of Sec. 3.9. Here, load current contains both positive and negative half-cycles. An analysis identical to that done for the controlled half-wave rectifier can be done on a half-cycle for the voltage controller. Then, by symmetry, the result can be extrapolated to describe the operation for the entire period.

Some basic observations about the circuit of Fig. 5-1*a* are as follows:

1. The SCRs cannot conduct simultaneously.
2. The load voltage is the same as the source voltage when either SCR is on. The load voltage is zero when both SCRs are off.
3. The switch voltage v_{sw} is zero when either SCR is on and is equal to the source voltage when neither is on.
4. The average current in the source and load is zero if the SCRs are on for equal time intervals. The average current in each SCR is not zero because of unidirectional SCR current.
5. The rms current in each SCR is $1/\sqrt{2}$ times the rms load current if the SCRs are on for equal time intervals. (Refer to Chap. 2.)

For the circuit of Fig. 5-1*a*, S_1 conducts if a gate signal is applied during the positive half-cycle of the source. Just as in the case of the SCR in the controlled half-wave rectifier, S_1 conducts until the current in it reaches zero. Where this circuit differs from the controlled half-wave rectifier is when the source is in its negative half-cycle. A gate signal is applied to S_2 during the negative half-cycle of the source, providing a path for negative load current. If the gate signal for S_2 is a half period later than that of S_1, analysis for the negative half-cycle is identical to that for the positive half, except for algebraic sign for the voltage and current.

Single-Phase Controller with a Resistive Load

Figure 5-1*b* shows the voltage waveforms for a single-phase phase-controlled voltage controller with a resistive load. These are the types of waveforms that exist in a common incandescent light-dimmer circuit. Let the source voltage be

$$v_s(\omega t) = V_m \sin \omega t \tag{5-1}$$

Output voltage is

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } \alpha < \omega t < \pi \text{ and } \alpha + \pi < \omega t < 2\pi \\ 0 & \text{otherwise} \end{cases} \tag{5-2}$$

The rms load voltage is determined by taking advantage of positive and negative symmetry of the voltage waveform, necessitating evaluation of only a half-period of the waveform:

$$V_{o,\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} \tag{5-3}$$

Note that for $\alpha = 0$, the load voltage is a sinusoid that has the same rms value as the source. Normalized rms load voltage is plotted as a function of α in Fig. 5-2.

The rms current in the load and the source is

$$I_{o,\text{rms}} = \frac{V_{o,\text{rms}}}{R} \tag{5-4}$$

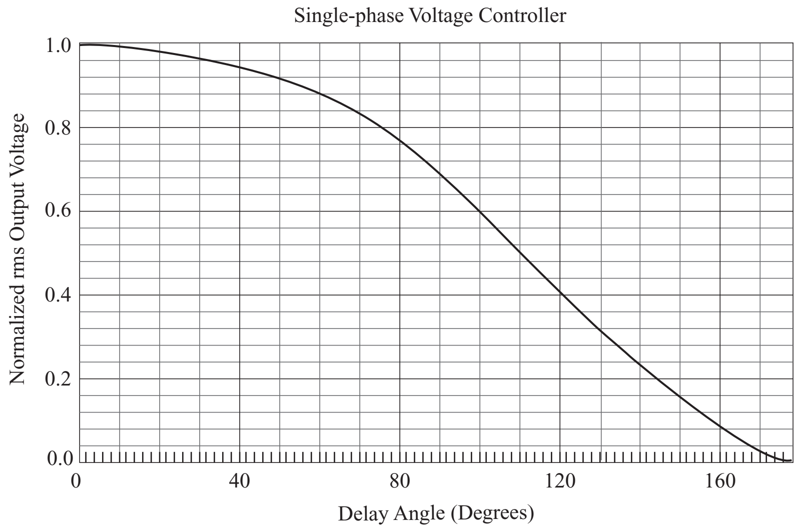

Figure 5-2 Normalized rms load voltage vs. delay angle for a single-phase ac voltage controller with a resistive load.

and the power factor of the load is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{V_{o,\text{rms}}^2 / R}{V_{s,\text{rms}}(V_{o,\text{rms}}/R)} = \frac{V_{o,\text{rms}}}{V_{s,\text{rms}}}$$

$$= \frac{\dfrac{V_m}{\sqrt{2}}\sqrt{1 - \dfrac{\alpha}{\pi} + \dfrac{(\sin 2\alpha)}{2\pi}}}{V_m/\sqrt{2}}$$

$$\boxed{\text{pf} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}} \tag{5-5}$$

Note that pf = 1 for $\alpha = 0$, which is the same as for an uncontrolled resistive load, and the power factor for $\alpha > 0$ is less than 1.

The average source current is zero because of half-wave symmetry. The average SCR current is

$$I_{\text{SCR,avg}} = \frac{1}{2\pi}\int_{\alpha}^{\pi} \frac{V_m \sin(\omega t)}{R}\, d(\omega t) = \frac{V_m}{2\pi R}(1 + \cos\alpha) \tag{5-6}$$

Since each SCR carries one-half of the line current, the rms current in each SCR is

$$I_{\text{SCR,rms}} = \frac{I_{o,\text{rms}}}{\sqrt{2}} \tag{5-7}$$

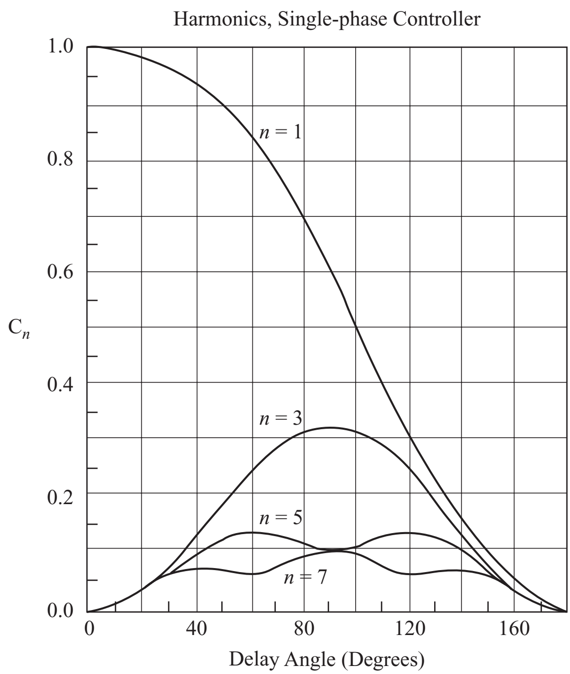

Figure 5-3 Normalized harmonic content vs. delay angle for a single-phase ac voltage controller with a resistive load; C_n is the normalized amplitude. (See Chap. 2.)

Since the source and load current is nonsinusoidal, harmonic distortion is a consideration when designing and applying ac voltage controllers. Only odd harmonics exist in the line current because the waveform has half-wave symmetry. Harmonic currents are derived from the defining Fourier equations in Chap. 2. Normalized harmonic content of the line currents vs. α is shown in Fig. 5-3. Base current is source voltage divided by resistance, which is the current for $\alpha = 0$.

EXAMPLE 5-1

Single-Phase Controller with a Resistive Load

The single-phase ac voltage controller of Fig. 5-1*a* has a 120-V rms 60-Hz source. The load resistance is 15 Ω. Determine (*a*) the delay angle required to deliver 500 W to the load, (*b*) the rms source current, (*c*) the rms and average currents in the SCRs, (*d*) the power factor, and (*e*) the total harmonic distortion (THD) of the source current.

■ Solution

(*a*) The required rms voltage to deliver 500 W to a 15-Ω load is

$$P = \frac{V_{o,\text{rms}}^2}{R}$$

$$V_{o,\text{rms}} = \sqrt{PR} = \sqrt{(500)(15)} = 86.6 \text{ V}$$

The relationship between output voltage and delay angle is described by Eq. (5-3) and Fig. 5-2. From Fig. 5-2, the delay angle required to obtain a normalized output of 86.6/120 = 0.72 is approximately 90°. A more precise solution is obtained from the numerical solution for α in Eq. (5-3), expressed as

$$86.6 - 120\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} = 0$$

which yields

$$\alpha = 1.54 \text{ rad} = 88.1°$$

(*b*) Source rms current is

$$I_{o,\text{rms}} = \frac{V_{o,\text{rms}}}{R} = \frac{86.6}{15} = 5.77 \text{ A}$$

(*c*) SCR currents are determined from Eqs. (5-6) and (5-7),

$$I_{\text{SCR,rms}} = \frac{I_{\text{rms}}}{\sqrt{2}} = \frac{5.77}{\sqrt{2}} = 4.08 \text{ A}$$

$$I_{\text{SCR,avg}} = \frac{\sqrt{2}(120)}{2\pi(15)}\left[1 + \cos(88.1°)\right] = 1.86 \text{ A}$$

(*d*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{500}{(120)(5.77)} = 0.72$$

which could also be computed from Eq. (5-5).

(*e*) Base rms current is

$$I_{\text{base}} = \frac{V_{s,\text{rms}}}{R} = \frac{120}{15} = 8.0 \text{ A}$$

The rms value of the current's fundamental frequency is determined from C_1 in the graph of Fig. 5-3.

$$C_1 \approx 0.61 \Rightarrow I_{1,\text{rms}} = C_1 I_{\text{base}} = (0.61)(8.0) = 4.9 \text{ A}$$

The THD is computed from Eq. (2-68),

$$\text{THD} = \frac{\sqrt{I_{\text{rms}}^2 - I_{1,\text{rms}}^2}}{I_{1,\text{rms}}} = \frac{\sqrt{5.77^2 - 4.9^2}}{4.9} = 0.63 = 63\%$$

Single-Phase Controller with an *RL* Load

Figure 5-4*a* shows a single-phase ac voltage controller with an *RL* load. When a gate signal is applied to S_1 at $\omega t = \alpha$, Kirchhoff's voltage law for the circuit is expressed as

$$V_m \sin(\omega t) = Ri_o(t) + L\frac{di_o(t)}{dt} \tag{5-8}$$

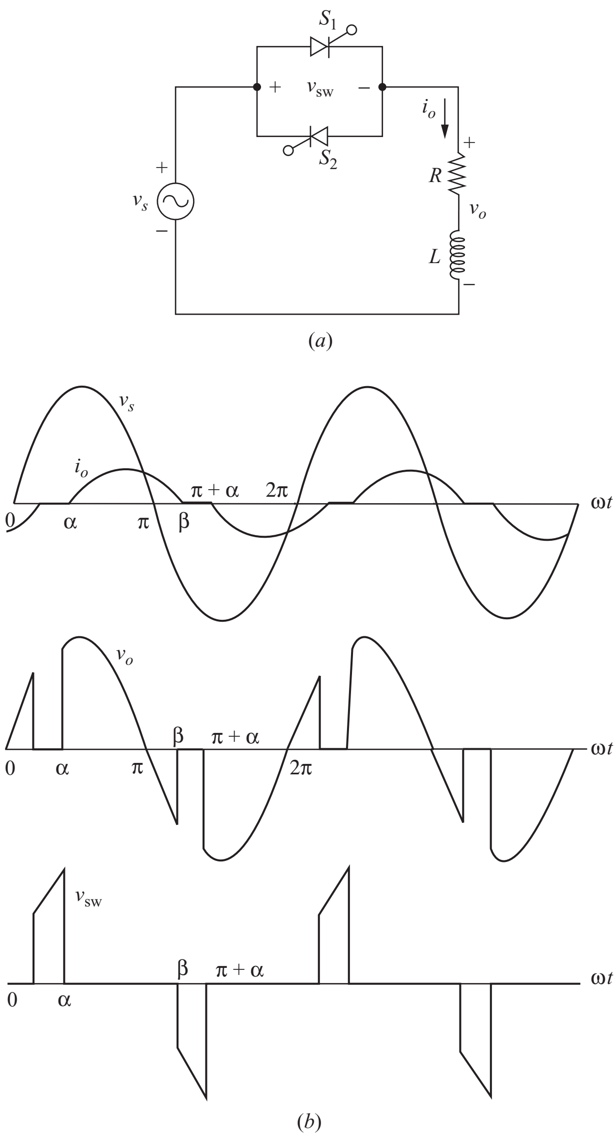

Figure 5-4 (*a*) Single-phase ac voltage controller with an *RL* load; (*b*) Typical waveforms.

The solution for current in this equation, outlined in Sec. 3.9, is

$$i_o(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where (5-9)

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The extinction angle β is the angle at which the current returns to zero, when $\omega t = \beta$,

$$i_o(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)\, e^{(\alpha - \beta)/\omega\tau}\right] \tag{5-10}$$

which must be solved numerically for β.

A gate signal is applied to S_2 at $\omega t = \pi + \alpha$, and the load current is negative but has a form identical to that of the positive half-cycle. Figure 5-4*b* shows typical waveforms for a single-phase ac voltage controller with an *RL* load. The conduction angle γ is defined as

$$\gamma = \beta - \alpha \tag{5-11}$$

In the interval between π and β when the source voltage is negative and the load current is still positive, S_2 cannot be turned on because it is not forward-biased. The gate signal to S_2 must be delayed at least until the current in S_1 reaches zero, at $\omega t = \beta$. The delay angle is therefore at least $\beta - \pi$.

$$\alpha \ge \beta - \pi \tag{5-12}$$

The limiting condition when $\beta - \alpha = \pi$ is determined from an examination of Eq. (5-10). When $\alpha = \theta$, Eq. (5-10) becomes

$$\sin(\beta - \alpha) = 0$$

which has a solution

$$\beta - \alpha = \pi$$

Therefore,

$$\gamma = \pi \qquad \text{when} \quad \alpha = \theta \tag{5-13}$$

If $\alpha < \theta$, $\gamma = \pi$, provided that the gate signal is maintained beyond $\omega t = \theta$.

In the limit, when $\gamma = \pi$, one SCR is always conducting, and the voltage across the load is the same as the voltage of the source. The load voltage and current are sinusoids for this case, and the circuit is analyzed using phasor analysis for ac circuits. *The power delivered to the load is continuously controllable between the two extremes corresponding to full source voltage and zero.*

This SCR combination can act as a *solid-state relay*, connecting or disconnecting the load from the ac source by gate control of the SCRs. The load is disconnected from the source when no gate signal is applied, and the load has the same voltage as the source when a gate signal is continuously applied. In practice, the gate signal may be a high-frequency series of pulses rather than a continuous dc signal.

An expression for rms load current is determined by recognizing that the square of the current waveform repeats every π rad. Using the definition of rms,

$$I_{o,\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o^2(\omega t)\, d(\omega t)} \tag{5-14}$$

where $i_o(\omega t)$ is described in Eq. (5-9).

Power absorbed by the load is determined from

$$P = I_{o,\text{rms}}^2 R \tag{5-15}$$

The rms current in each SCR is

$$I_{\text{SCR,rms}} = \frac{I_{o,\text{rms}}}{\sqrt{2}} \tag{5-16}$$

The average load current is zero, but each SCR carries one-half of the current waveform, making the average SCR current

$$I_{\text{SCR,avg}} = \frac{1}{2\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t) \tag{5-17}$$

EXAMPLE 5-2

Single-Phase Voltage Controller with *RL* Load

For the single-phase voltage controller of Fig. 5-4*a*, the source is 120 V rms at 60 Hz, and the load is a series *RL* combination with $R = 20\ \Omega$ and $L = 50$ mH. The delay angle α is 90°. Determine (*a*) an expression for load current for the first half-period, (*b*) the rms load current, (*c*) the rms SCR current, (*d*) the average SCR current, (*e*) the power delivered to the load, and (*f*) the power factor.

■ Solution

(*a*) The current is expressed as in Eq. (5-9). From the parameters given,

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{(20)^2 + \left[(377)(0.05)\right]^2} = 27.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\frac{(377)(0.05)}{20} = 0.756 \text{ rad}$$

$$\omega\tau = \omega\left(\frac{L}{R}\right) = 377\left(\frac{0.05}{20}\right) = 0.943 \text{ rad}$$

$$\frac{V_m}{Z} = \frac{120\sqrt{2}}{27.5} = 6.18 \text{ A}$$

$$\alpha = 90° = 1.57 \text{ rad}$$

$$\frac{V_m}{Z}\sin(\alpha-\theta)\,e^{\alpha/\omega\tau}=23.8\text{ A}$$

The current is then expressed in Eq. (5-9) as

$$i_o(\omega t)=6.18\sin(\omega t-0.756)-23.8e^{-\omega t/0.943}\quad\text{A}\qquad\text{for } \alpha\le\omega t\le\beta$$

The extinction angle β is determined from the numerical solution of $i(\beta)=0$ in the above equation, yielding

$$\beta=3.83\text{ rad}=220°$$

Note that the conduction angle $\gamma=\beta-\alpha=2.26\text{ rad}=130°$, which is less than the limit of 180°.

(*b*) The rms load current is determined from Eq. (5-14).

$$I_{o,\text{rms}}=\sqrt{\frac{1}{\pi}\int_{1.57}^{3.83}\left[6.18\sin(\omega t-0.756)-23.8e^{-\omega t/0.943}\right]d(\omega t)}=2.71\text{ A}$$

(*c*) The rms current in each SCR is determined from Eq. (5-16).

$$I_{\text{SCR,rms}}=\frac{I_{o,\text{rms}}}{\sqrt{2}}=\frac{2.71}{\sqrt{2}}=1.92\text{ A}$$

(*d*) Average SCR current is obtained from Eq. (5-17).

$$I_{\text{SCR, avg}}=\frac{1}{2\pi}\int_{1.57}^{3.83}\left[6.18\sin(\omega t-0.756)-23.8e^{-\omega t/0.943}\right]d(\omega t)=1.04\text{ A}$$

(*e*) Power absorbed by the load is

$$P=I_{o,\text{rms}}^2R=(2.71)^2(20)=147\text{ W}$$

(*f*) Power factor is determined from *P*/*S*.

$$\text{pf}=\frac{P}{S}=\frac{P}{V_{s,\text{rms}}I_{s,\text{rms}}}=\frac{147}{(120)(2.71)}=0.45=45\%$$