## 3.8 HALF-WAVE RECTIFIER WITH A CAPACITOR FILTER

### Creating a DC Voltage from an AC Source

A common application of rectifier circuits is to convert an ac voltage input to a dc voltage output. The half-wave rectifier of Fig. 3-11*a* has a parallel *RC* load. The purpose of the capacitor is to reduce the variation in the output voltage, making it more like dc. The resistance may represent an external load, and the capacitor may be a filter which is part of the rectifier circuit.

Assuming the capacitor is initially uncharged and the circuit is energized at $\omega t = 0$, the diode becomes forward-biased as the source becomes positive. With the diode on, the output voltage is the same as the source voltage, and the capacitor charges. The capacitor is charged to $V_m$ when the input voltage reaches its positive peak at $\omega t = \pi/2$.

As the source decreases after $\omega t = \pi/2$, the capacitor discharges into the load resistor. At some point, the voltage of the source becomes less than the output voltage, reverse-biasing the diode and isolating the load from the source. The output voltage is a decaying exponential with time constant *RC* while the diode is off.

The point when the diode turns off is determined by comparing the rates of change of the source and the capacitor voltages. The diode turns off when the

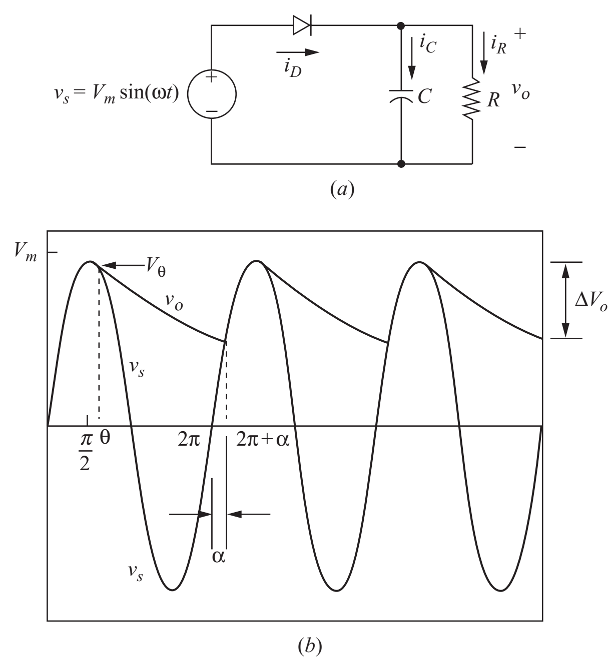


**Figure 3-11** (*a*) Half-wave rectifier with *RC* load; (*b*) Input and output voltages.

downward rate of change of the source exceeds that permitted by the time constant of the $RC$ load. The angle $\omega t = \theta$ is the point when the diode turns off in Fig. 3-11$b$. The output voltage is described by

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases} \tag{3-37}$$

where

$$V_\theta = V_m \sin \theta \tag{3-38}$$

The slopes of these functions are

$$\frac{d}{d(\omega t)}[V_m \sin(\omega t)] = V_m \cos(\omega t) \tag{3-39}$$

and

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta\, e^{-(\omega t - \theta)/\omega RC}\right) = V_m \sin\theta\left(-\frac{1}{\omega RC}\right)e^{-(\omega t - \theta)/\omega RC} \tag{3-40}$$

At $\omega t = \theta$, the slopes of the voltage functions are equal:

$$V_m \cos\theta = \left(\frac{V_m \sin\theta}{-\omega RC}\right)e^{-(\theta-\theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\frac{V_m \cos\theta}{V_m \sin\theta} = \frac{1}{-\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

Solving for $\theta$ and expressing $\theta$ so it is in the proper quadrant, we have

$$\boxed{\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi} \tag{3-41}$$

In practical circuits where the time constant is large,

$$\boxed{\theta \approx \frac{\pi}{2} \qquad \text{and} \qquad V_m \sin\theta \approx V_m} \tag{3-42}$$

When the source voltage comes back up to the value of the output voltage in the next period, the diode becomes forward-biased, and the output again is the same as the source voltage. The angle at which the diode turns on in the second period, $\omega t = 2\pi + \alpha$, is the point when the sinusoidal source reaches the same value as the decaying exponential output:

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta)e^{-(2\pi + \alpha - \theta)/\omega RC}$$

or

$$\boxed{\sin\alpha - (\sin\theta)e^{-(2\pi+\alpha-\theta)/\omega RC} = 0} \tag{3-43}$$

Equation (3-43) must be solved numerically for $\alpha$.

The current in the resistor is calculated from $i_R = v_o/R$. The current in the capacitor is calculated from

$$i_C(t) = C\frac{dv_o(t)}{dt}$$

which can also be expressed, using $\omega t$ as the variable, as

$$i_C(\omega t) = \omega C\frac{dv_o(\omega t)}{d(\omega t)}$$

Using $v_o$ from Eq. (3-37),

$$\boxed{i_C(\omega t) = \begin{cases} -\left(\dfrac{V_m\sin\theta}{R}\right)e^{-(\omega t-\theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi+\alpha \quad \text{(diode off)} \\ \omega C V_m\cos(\omega t) & \text{for } 2\pi+\alpha \le \omega t \le 2\pi+\theta \quad \text{(diode on)} \end{cases}} \tag{3-44}$$

The source current, which is the same as the diode current, is

$$i_S = i_D = i_R + i_C \tag{3-45}$$

The average capacitor current is zero, so the average diode current is the same as the average load current. Since the diode is on for a short time in each cycle, the peak diode current is generally much larger than the average diode current. Peak capacitor current occurs when the diode turns on at $\omega t = 2\pi + \alpha$. From Eq. (3-44),

$$I_{C,\text{peak}} = \omega C V_m\cos(2\pi+\alpha) = \omega C V_m\cos\alpha \tag{3-46}$$

Resistor current at $\omega t = 2\pi + \alpha$ is obtained from Eq. (3-37).

$$i_R(2\omega t + \alpha) = \frac{V_m\sin(2\omega t+\alpha)}{R} = \frac{V_m\sin\alpha}{R} \tag{3-47}$$

Peak diode current is

$$\boxed{I_{D,\text{peak}} = \omega C V_m\cos\alpha + \frac{V_m\sin\alpha}{R} = V_m\left(\omega C\cos\alpha + \frac{\sin\alpha}{R}\right)} \tag{3-48}$$

The effectiveness of the capacitor filter is determined by the variation in output voltage. This may be expressed as the difference between the maximum and minimum output voltage, which is the peak-to-peak ripple voltage. For the half-wave rectifier of Fig. 3-11*a*, the maximum output voltage is $V_m$. The minimum output voltage occurs at $\omega t = 2\pi + \alpha$, which can be computed from $V_m \sin \alpha$. The peak-to-peak ripple for the circuit of Fig. 3-11*a* is expressed as

$$\boxed{\Delta V_o = V_m - V_m \sin \alpha = V_m(1 - \sin \alpha)} \tag{3-49}$$

In circuits where the capacitor is selected to provide for a nearly constant dc output voltage, the *RC* time constant is large compared to the period of the sine wave, and Eq. (3-42) applies. Moreover, the diode turns on close to the peak of the sine wave when $\alpha \approx \pi/2$. The change in output voltage when the diode is off is described in Eq. (3-37). In Eq. (3-37), if $V_\theta \approx V_m$ and $\theta \approx \pi/2$, then Eq. (3-37) evaluated at $\alpha = \pi/2$ is

$$v_o(2\pi + \alpha) = V_m e^{-(2\pi + \pi/2 - \pi/2)\omega RC} = V_m e^{-2\pi/\omega RC}$$

The ripple voltage can then be approximated as

$$\Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m\left(1 - e^{-2\pi/\omega RC}\right) \tag{3-50}$$

Furthermore, the exponential in the above equation can be approximated by the series expansion:

$$e^{-2\pi/\omega RC} \approx 1 - \frac{2\pi}{\omega RC}$$

Substituting for the exponential in Eq. (3-50), the peak-to-peak ripple is approximately

$$\boxed{\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}} \tag{3-51}$$

The output voltage ripple is reduced by increasing the filter capacitor *C*. As *C* increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.

**EXAMPLE 3-9**

### Half-Wave Rectifier with *RC* Load

The half-wave rectifier of Fig. 3-11*a* has a 120-V rms source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu$F. Determine (*a*) an expression for output voltage, (*b*) the peak-to-peak voltage variation on the output, (*c*) an expression for capacitor current, (*d*) the peak diode current, and (*e*) the value of *C* such that $\Delta V_o$ is 1 percent of $V_m$.

#### ■ Solution

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85 \text{ rad}$$

The angle $\theta$ is determined from Eq. (3-41).

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62\,\text{rad} = 93°$$

$$V_m \sin\theta = 169.5 \text{ V}$$

The angle $\alpha$ is determined from the numerical solution of Eq. (3-43).

$$\sin\alpha - \sin(1.62)e^{-(2\pi + \alpha - 1.62/18.85)} = 0$$

yielding

$$\alpha = 0.843 \text{ rad} = 48°$$

(*a*) Output voltage is expressed from Eq. (3-37).

$$v_o(\omega t) = \begin{cases} 169.7 \sin(\omega t) & 2\pi + \alpha \le \omega t \le 2\pi + \theta \\ 169.5e^{-(\omega t - 1.62)/18.85} & \theta \le \omega t \le 2\pi + \alpha \end{cases}$$

(*b*) Peak-to-peak output voltage is described by Eq. (3-49).

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7(1 - \sin 0.843) = 43 \text{ V}$$

(*c*) The capacitor current is determined from Eq. (3-44).

$$i_C(\omega t) = \begin{cases} -0.339e^{-(\omega t - 1.62)/18.85} \text{ A} & \theta \le \omega t \le 2\pi + \alpha \\ 6.4\cos(\omega t) \text{ A} & 2\pi + \alpha \le \omega t \le 2\pi + \theta \end{cases}$$

(*d*) Peak diode current is determined from Eq. (3-48).

$$I_{D,\text{peak}} = \sqrt{2}(120)\left[377(10)^{-4}\cos 0.843 + \frac{\sin 8.43}{500}\right]$$
$$= 4.26 + 0.34 = 4.50 \text{ A}$$

(*e*) For $\Delta V_o = 0.01V_m$, Eq. (3-51) can be used.

$$C \approx \frac{V_m}{fR(\Delta V_o)} = \frac{V_m}{(60)(500)(0.01V_m)} = \frac{1}{300}\text{F} = 3333\ \mu\text{F}$$

Note that peak diode current can be determined from Eq. (3-48) using an estimate of $\alpha$ from Eq. (3-49).

$$\alpha \approx \sin^{-1}\left(1 - \frac{\Delta V_o}{V_m}\right) = \sin^{-1}\left(1 - \frac{1}{fRC}\right) = 81.9°$$

From Eq. (3-48), peak diode current is 30.4 A.

the period of the input voltage, resulting in little decay of the output voltage. For an effective filter capacitor, the output voltage is essentially the same as the peak voltage of the input.

## 3.9 THE CONTROLLED HALF-WAVE RECTIFIER

The half-wave rectifiers analyzed previously in this chapter are classified as uncontrolled rectifiers. Once the source and load parameters are established, the dc level of the output and the power transferred to the load are fixed quantities.

A way to control the output of a half-wave rectifier is to use an SCR[1] instead of a diode. Figure 3-13*a* shows a basic controlled half-wave rectifier with a resistive load. Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($v_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.

Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control. Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.

### Resistive Load

Figure 3-13*b* shows the voltage waveforms for a controlled half-wave rectifier with a resistive load. A gate signal is applied to the SCR at $\omega t = \alpha$, where $\alpha$ is the delay angle. The average (dc) voltage across the load resistor in Fig. 3-13*a* is

$$\boxed{V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)} \tag{3-52}$$

The power absorbed by the resistor is $V_{\text{rms}}^2/R$, where the rms voltage across the resistor is computed from

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v_o^2(\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2\, d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} \tag{3-53}$$

[1] Switching with other controlled turn-on devices such as transistors or IGBTs can be used to control the output of a converter.

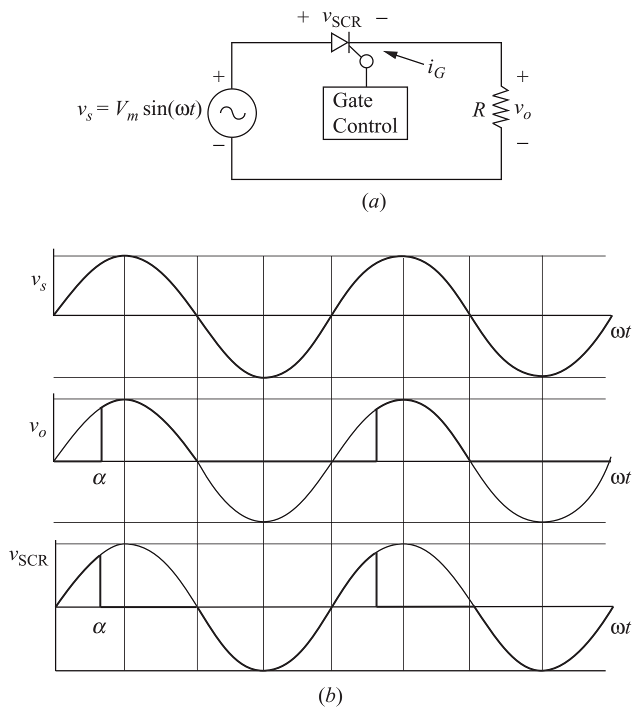


**Figure 3-13** (*a*) A basic controlled rectifier; (*b*) Voltage waveforms.

**EXAMPLE 3-10**

## Controlled Half-Wave Rectifier with Resistive Load

Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

### ■ Solution

If an uncontrolled half-wave rectifier is used, the average voltage will be $V_m/\pi = 120\sqrt{2}/\pi = 54$ V. Some means of reducing the average resistor voltage to the design specification of 40 V must be found. A series resistance or inductance could be added to an uncontrolled rectifier, or a controlled rectifier could be used. The controlled rectifier of Fig. 3-13*a* has the advantage of not altering the load or introducing losses, so it is selected for this application.

Equation (3-52) is rearranged to determine the required delay angle:

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right]$$

$$= \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

Equation (3-53) gives

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

Load power is

$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

The power factor of the circuit is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

---

## *RL* Load

A controlled half-wave rectifier with an *RL* load is shown in Fig. 3-14*a*. The analysis of this circuit is similar to that of the uncontrolled rectifier. The current is the sum of the forced and natural responses, and Eq. (3-9) applies:

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant $A$ is determined from the initial condition $i(\alpha) = 0$:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right] = e^{\alpha/\omega\tau} \qquad \text{(3-54)}$$

Substituting for $A$ and simplifying,

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases} \qquad \text{(3-55)}$$

The *extinction angle* β is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When $\omega t = \beta$,

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega\tau}\right] \qquad \text{(3-56)}$$

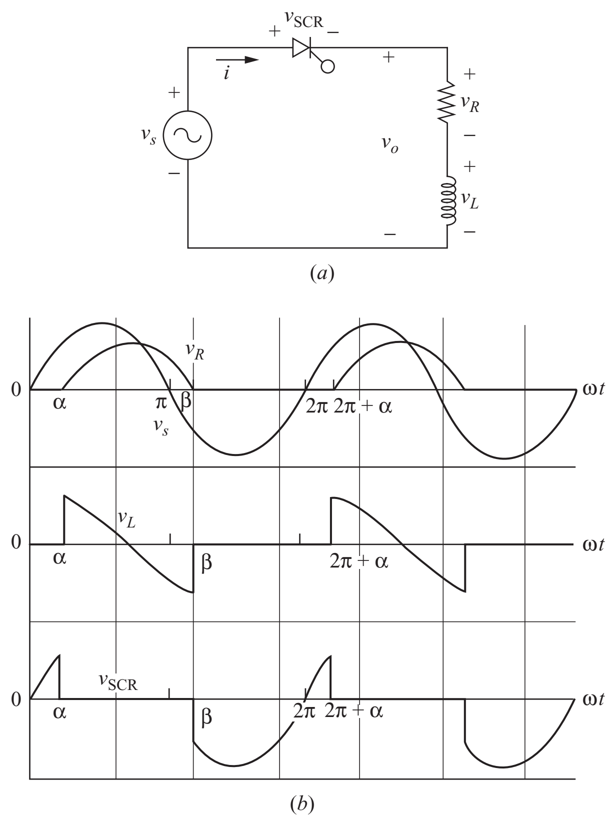


**Figure 3-14** (*a*) Controlled half-wave rectifier with *RL* load; (*b*) Voltage waveforms.

which must be solved numerically for β. The angle β − α is called the *conduction angle* γ. Figure 3-14*b* shows the voltage waveforms.

The average (dc) output voltage is

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) \tag{3-57}$$

The average current is computed from

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) \tag{3-58}$$

where $i(\omega t)$ is defined in Eq. (3-55). Power absorbed by the load is $I_{rms}^2 R$, where the rms current is computed from

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} \tag{3-59}$$

**EXAMPLE 3-11**

## Controlled Half-Wave Rectifier with *RL* Load

For the circuit of Fig. 3-14*a*, the source is 120 V rms at 60 Hz, $R = 20\ \Omega$, $L = 0.04$ H, and the delay angle is 45°. Determine (*a*) an expression for $i(\omega t)$, (*b*) the average current, (*c*) the power absorbed by the load, and (*d*) the power factor.

**■ Solution**

(*a*) From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [20^2 + (377*0.04)^2]^{0.5} = 25.0\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.04/20) = 0.646 \text{ rad}$$
$$\omega\tau = \omega L/R = 377*0.04/20 = 0.754$$
$$\alpha = 45° = 0.785 \text{ rad}$$

Substituting the preceding quantities into Eq. (3-55), current is expressed as

$$i(\omega t) = 6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754} \quad \text{A} \qquad \text{for } \alpha \leq \omega t \leq \beta$$

The preceding equation is valid from $\alpha$ to $\beta$, where $\beta$ is found numerically by setting the equation to zero and solving for $\omega t$, with the result $\beta = 3.79$ rad (217°). The conduction angle is $\gamma = \beta - \alpha = 3.79 - 0.785 = 3.01$ rad $= 172°$.

(*b*) Average current is determined from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]d(\omega t) = 2.19 \text{ A}$$

(*c*) The power absorbed by the load is computed from $I^2_{\text{rms}}R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]^2 d(\omega t)} = 3.26 \text{ A}$$

yielding

$$P = I^2_{\text{rms}}R = (3.26)^2(20) = 213 \text{ W}$$

(*d*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{213}{(120)(3.26)} = 0.54$$

### *RL*-Source Load

A controlled rectifier with a series resistance, inductance, and dc source is shown in Fig. 3-15. The analysis of this circuit is very similar to that of the uncontrolled half-wave rectifier discussed earlier in this chapter. The major difference is that for the uncontrolled rectifier, conduction begins as soon as the source voltage

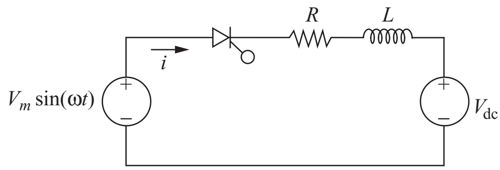


**Figure 3-15** Controlled rectifier with *RL*-source load.

reaches the level of the dc voltage. For the controlled rectifier, conduction begins when a gate signal is applied to the SCR, provided the SCR is forward-biased. Thus, the gate signal may be applied at any time that the ac source is larger than the dc source:

$$\alpha_{\min} = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) \tag{3-60}$$

Current is expressed as in Eq. (3-22), with α specified within the allowable range:

$$i(\omega t)\begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases} \tag{3-61}$$

where *A* is determined from Eq. (3-61):

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

**EXAMPLE 3-12**

## Controlled Rectifier with *RL*-Source Load

The controlled half-wave rectifier of Fig. 3-15 has an ac input of 120 V rms at 60 Hz, $R = 2\ \Omega$, $L = 20$ mH, and $V_{dc} = 100$ V. The delay angle α is 45°. Determine (*a*) an expression for the current, (*b*) the power absorbed by the resistor, and (*c*) the power absorbed by the dc source in the load.

■ **Solution:**

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [2^2 + (377*0.02)^2]^{0.5} = 7.80\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.02)/2) = 1.312 \text{ rad}$$
$$\omega\tau = \omega L/R = 377*0.02/2 = 3.77$$
$$\alpha = 45^\circ = 0.785 \text{ rad}$$

(*a*) First, use Eq. (3-60) to determine if $\alpha = 45°$ is allowable. The minimum delay angle is

$$\alpha_{\min} = \sin^{-1}\left(\frac{100}{120\sqrt{2}}\right) = 36°$$

which indicates that 45° is allowable. Equation (3-61) becomes

$$i(\omega t) = 21.8 \sin(\omega t - 1.312) - 50 + 75.0e^{-\omega t/3.77} \text{ A} \quad \text{for } 0.785 \leq \omega t \leq 3.37 \text{ rad}$$

where the extinction angle $\beta$ is found numerically to be 3.37 rad from the equation $i(\beta) = 0$.

(*b*) Power absorbed by the resistor is $I^2_{\text{rms}}R$, where $I_{\text{rms}}$ is computed from Eq. (3-59) using the preceding expression for $i(\omega t)$.

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\,d(\omega t)} = 3.90 \text{ A}$$

$$P = (3.90)^2(2) = 30.4 \text{ W}$$

(*c*) Power absorbed by the dc source is $I_o V_{\text{dc}}$, where $I_o$ is computed from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\,d(\omega t) = 2.19 \text{ A}$$

$$P_{\text{dc}} = I_o V_{\text{dc}} = (2.19)(100) = 219 \text{ W}$$

The following results are obtained from Probe for $\alpha = 70°$:

| Quantity | Expression | Result |
|---|---|---|
| DC source power | AVG(W(Vdc)) | 148 W (design objective of 150 W) |
| RMS current | RMS(I(R1)) | 2.87 A |
| Resistor power | AVG(W(R1)) | 16.5 W |
| Source apparent power | RMS(V(SOURCE))*RMS(I(Vs)) | 344 VA |
| Source average power | AVG(W(Vs)) | 166 W |
| Power factor ($P/S$) | 166/344 | 0.48 |

## 3.11 COMMUTATION

### The Effect of Source Inductance

The preceding discussion on half-wave rectifiers assumed an ideal source. In practical circuits, the source has an equivalent impedance which is predominantly inductive reactance. For the single-diode half-wave rectifiers of Figs. 3-1 and 3-2, the nonideal circuit is analyzed by including the source inductance with the load elements. However, the source inductance causes a fundamental change in circuit behavior for circuits like the half-wave rectifier with a freewheeling diode.

A half-wave rectifier with a freewheeling diode and source inductance $L_s$ is shown in Fig. 3-18*a*. Assume that the load inductance is very large, making the load current constant. At $t = 0^-$, the load current is $I_L$, $D_1$ is off, and $D_2$ is on. As the source voltage becomes positive, $D_1$ turns on, but the source current does not instantly equal the load current because of $L_s$. Consequently, $D_2$ must remain on while the current in $L_s$ and $D_1$ increases to that of the load. The interval when both $D_1$ and $D_2$ are on is called the commutation time or commutation angle. *Commutation is the process of turning off an electronic switch, which usually involves transferring the load current from one switch to another.*[2]

When both $D_1$ and $D_2$ are on, the voltage across $L_s$ is

$$v_{Ls} = V_m \sin(\omega t) \tag{3-62}$$

and current in $L_s$ and the source is

$$i_s = \frac{1}{\omega L_s}\int_0^{\omega t} v_{Ls}\,d(\omega t) + i_s(0) = \frac{1}{\omega L_s}\int_0^{\omega t} V_m \sin(\omega t)\,d(\omega t) + 0$$
$$i_s = \frac{V_m}{\omega L_s}(1 - \cos\omega t) \tag{3-63}$$

[2] Commutation in this case is an example of *natural commutation* or *line commutation,* where the change in instantaneous line voltage results in a device turning off. Other applications may use *forced commutation,* where current in a device such as a thyristor is forced to zero by additional circuitry. *Load commutation* makes use of inherent oscillating currents produced by the load to turn a device off.

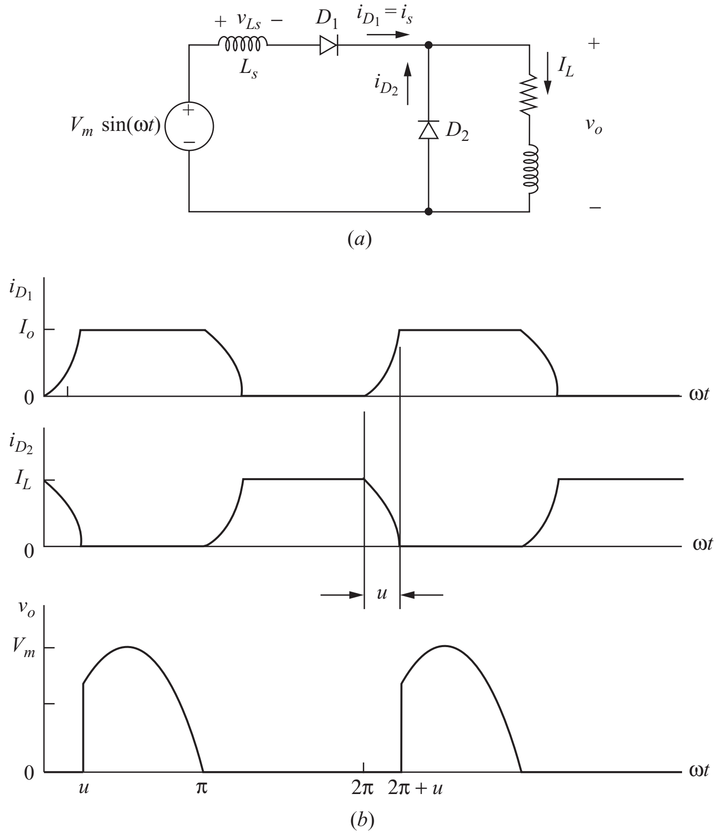


**Figure 3-18** (*a*) Half-wave rectifier with freewheeling diode and source inductance; (*b*) Diode currents and load voltage showing the effects of Commutation.

Current in $D_2$ is

$$i_{D_2} = I_L - i_s = I_L - \frac{V_m}{\omega L_s}(1 - \cos \omega t)$$

The current in $D_2$ starts at $I_L$ and decreases to zero. Letting the angle at which the current reaches zero be $\omega t = u$,

$$i_{D_2}(u) = I_L - \frac{V_m}{\omega L_s}(1 - \cos u) = 0$$

Solving for $u$,

$$\boxed{u = \cos^{-1}\left(1 - \frac{I_L \omega L_s}{V_m}\right) = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right)} \quad (3\text{-}64)$$

where $X_s = \omega L_s$ is the reactance of the source. Figure 3-18*b* shows the effect of the source reactance on the diode currents. The commutation from $D_1$ to $D_2$ is analyzed similarly, yielding an identical result for the commutation angle $u$.

The commutation angle affects the voltage across the load. Since the voltage across the load is zero when $D_2$ is conducting, the load voltage remains at zero through the commutation angle, as shown in Fig. 3-17*b*. Recall that the load voltage is a half-wave rectified sinusoid when the source is ideal.

Average load voltage is

$$V_o = \frac{1}{2\pi}\int_u^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{V_m}{2\pi}[-\cos(\omega t)]\Big|_u^{\pi} = \frac{V_m}{2\pi}(1 + \cos u)$$

Using $u$ from Eq. (3-64),

$$\boxed{V_o = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right)} \tag{3-65}$$

Recall that the average of a half-wave rectified sine wave is $V_m/\pi$. Source reactance thus reduces average load voltage.

## 3.12 Summary

- A rectifier converts ac to dc. Power transfer is from the ac source to the dc load.
- The half-wave rectifier with a resistive load has an average load voltage of $V_m/\pi$ and an average load current of $V_m/\pi R$.
- The current in a half-wave rectifier with an *RL* load contains a natural and a forced response, resulting in

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2}, \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{and} \quad \tau = \frac{L}{R}$$

The diode remains on as long as the current is positive. Power in the *RL* load is $I_{\text{rms}}^2 R$.

- A half-wave rectifier with an *RL*-source load does not begin to conduct until the ac source reaches the dc voltage in the load. Power in the resistance is $I_{\text{rms}}^2 R$, and power absorbed by the dc source is $I_o V_{\text{dc}}$, where $I_o$ is the average load current. The load current is expressed as

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where

$$A = \left[ -\frac{V_m}{Z} \sin(\alpha - \beta) + \frac{V_{dc}}{R} \right] e^{\alpha/\omega\tau}$$

- A freewheeling diode forces the voltage across an *RL* load to be a half-wave rectified sine wave. The load current can be analyzed using Fourier analysis. A large load inductance results in a nearly constant load current.
- A large filter capacitor across a resistive load makes the load voltage nearly constant. Average diode current must be the same as average load current, making the peak diode current large.
- An SCR in place of the diode in a half-wave rectifier provides a means of controlling output current and voltage.
- 

# Problems

## Half-Wave Rectifier with Resistive Load

**3-1.** The half-wave rectifier circuit of Fig. 3-1*a* has $v_s(t) = 170 \sin(377t)$ V and a load resistance $R = 15\ \Omega$. Determine (*a*) the average load current, (*b*) the rms load current, (*c*) the power absorbed by the load, (*d*) the apparent power supplied by the source, and (*e*) the power factor of the circuit.

**3-2.** The half-wave rectifier circuit of Fig. 3-1*a* has a transformer inserted between the source and the remainder of the circuit. The source is 240 V rms at 60 Hz, and the load resistor is 20 Ω. (*a*) Determine the required turns ratio of the transformer such that the average load current is 12 A. (*b*) Determine the average current in the primary winding of the transformer.

**3-3.** For a half-wave rectifier with a resistive load, (*a*) show that the power factor is 1/ and (*b*) determine the displacement power factor and the distortion factor as defined in Chap. 2. The Fourier series for the half-wave rectified voltage is given in Eq. (3-34).

$\sqrt{2}$

## Half-Wave Rectifier with *RL* Load

**3-4.** A half-wave rectifier has a source of 120 V rms at 60 Hz and an *RL* load with $R = 12\ \Omega$ and $L = 12$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor.

**3-5.** A half-wave rectifier has a source of 120 V rms at 60 Hz and an *RL* load with $R = 10\ \Omega$ and $L = 15$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor.

## Half-Wave Rectifier with *RL*-Source Load

**3-8.** A half-wave rectifier of Fig. 3-5*a* has a 240 V rms, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 75$ mH, $R = 10\ \Omega$, and $V_{dc} = 100$ V. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

**3-9.** A half-wave rectifier of Fig. 3-5*a* has a 120 V rms, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 120$ mH, $R = 12\ \Omega$, and $V_{dc} = 48$ V. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

**3-10.** A half-wave rectifier of Fig. 3-6 has a 120 V rms, 60 Hz ac source. The load is a series inductance and dc voltage with $L = 100$ mH and $V_{dc} = 48$ V. Determine the power absorbed by the dc voltage source.

## Freewheeling Diode

**3-13.** The half-wave rectifier with a freewheeling diode (Fig. 3-7*a*) has $R = 12\ \Omega$ and $L = 60$ mH. The source is 120 V rms at 60 Hz. (*a*) From the Fourier series of the half-wave rectified sine wave that appears across the load, determine the dc component of the current. (*b*) Determine the amplitudes of the first four nonzero ac terms in the Fourier series. Comment on the results.

**3-16.** The circuit of Fig. P3-16 is similar to the circuit of Fig. 3-7*a* except that a dc source has been added to the load. The circuit has $v_s(t) = 170 \sin(377t)$ V, $R = 10\ \Omega$, and $V_{dc} = 24$ V. From the Fourier series, (*a*) determine the value of $L$ such that the peak-to-peak variation in load current is no more than 1 A. (*b*) Determine the power absorbed by the dc source. (*c*) Determine the power absorbed by the resistor.

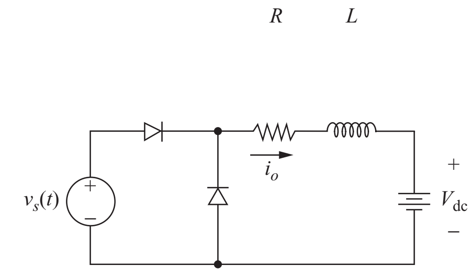


**Figure** P3-16

## Half-Wave Rectifier with a Filter Capacitor

**3-17.** A half-wave rectifier with a capacitor filter has $V_m = 200$ V, $R = 1$ kω, $C = 1000\ \mu$F, and $\omega = 377$. (*a*) Determine the ratio of the *RC* time constant to the period of the input sine wave. What is the significance of this ratio? (*b*) Determine the peak-to-peak ripple voltage using the exact equations. (*c*) Determine the ripple using the approximate formula in Eq. (3-51).

**3-18.** Repeat Prob. 3-17 with (*a*) $R = 100\ \Omega$ and (*b*) $R = 10\ \Omega$. Comment on the results.

**3-19.** A half-wave rectifier with a 1-kΩ load has a parallel capacitor. The source is 120 V rms, 60 Hz. Determine the peak-to-peak ripple of the output voltage when the capacitor is (*a*) 4000 μF and (*b*) 20 μF. Is the approximation of Eq. (3-51) reasonable in each case?

**3-20.** Repeat Prob. 3-19 with $R = 500\ \Omega$.

**3-21.** A half-wave rectifier has a 120 V rms, 60 Hz ac source. The load is 750 Ω. Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 2 V. Determine the average and peak values of diode current.

**3-22.** A half-wave rectifier has a 120 V rms 60 Hz ac source. The load is 50 W. (*a*) Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 1.5 V. (*b*) Determine the average and peak values of diode current.

## Controlled Half-Wave Rectifier

**3-23.** Show that the controlled half-wave rectifier with a resistive load in Fig. 3-13*a* has a power factor of

$$\text{pf} = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

**3-24.** For the controlled half-wave rectifier with resistive load, the source is 120 V rms at 60 Hz. The resistance is 100 Ω, and the delay angle α is 45°. (*a*) Determine the average voltage across the resistor. (*b*) Determine the power absorbed by the resistor. (*c*) Determine the power factor as seen by the source.

**3-25.** A controlled half-wave rectifier has an ac source of 240 V rms at 60 Hz. The load is a 30-Ω resistor. (*a*) Determine the delay angle such that the average load current is 2.5 A. (*b*) Determine the power absorbed by the load. (*c*) Determine the power factor.

**3-26.** A controlled half-wave rectifier has a 120 V rms 60 Hz ac source. The series *RL* load has $R = 25$ ω and $L = 50$ mH. The delay angle is 30°. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

**3-27.** A controlled half-wave rectifier has a 120 V rms 60 Hz ac source. The series *RL* load has $R = 40\ \Omega$ and $L = 75$ mH. The delay angle is 60°. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

**3-30.** A controlled half-wave rectifier has a 120 V, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 100$ mH, $R = 12\ \Omega$, and $V_{dc} = 48$ V. The delay angle is 50°. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

**3-31.** A controlled half-wave rectifier has a 240 V rms 60 Hz ac source. The load is a series resistance, inductance, and dc source with $R = 100\ \Omega$, $L = 150$ mH, and $V_{dc} = 96$ V. The delay angle is 60°. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

**3-36.** A controlled half-wave rectifier has an *RL* load. A freewheeling diode is placed in parallel with the load. The inductance is large enough to consider the load current to be constant. Determine the load current as a function of the delay angle alpha. Sketch the current in the SCR and the freewheeling diode. Sketch the voltage across the load.