Lecture 5: Dynamic Models

Block Diagram Model

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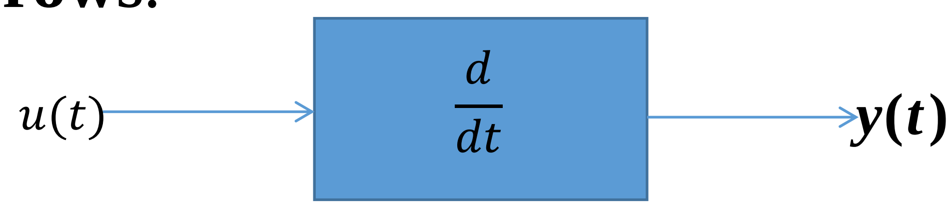
§ 5.1 The Block Diagram Model

- The Block diagram model is a graphical tool to **visualize** the model of a system and evaluate the mathematical relationships between its components, using their transfer functions.
- Block diagram is a shorthand, graphical representation of a physical system.
- In many control systems, the system equations can be written so that their components do not interact except by having **the input of one part be the output of another part**.
- The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the
- The transfer function of each components is placed in a **box**, and the input-output relationships between components are indicated by **lines** and **arrows**.

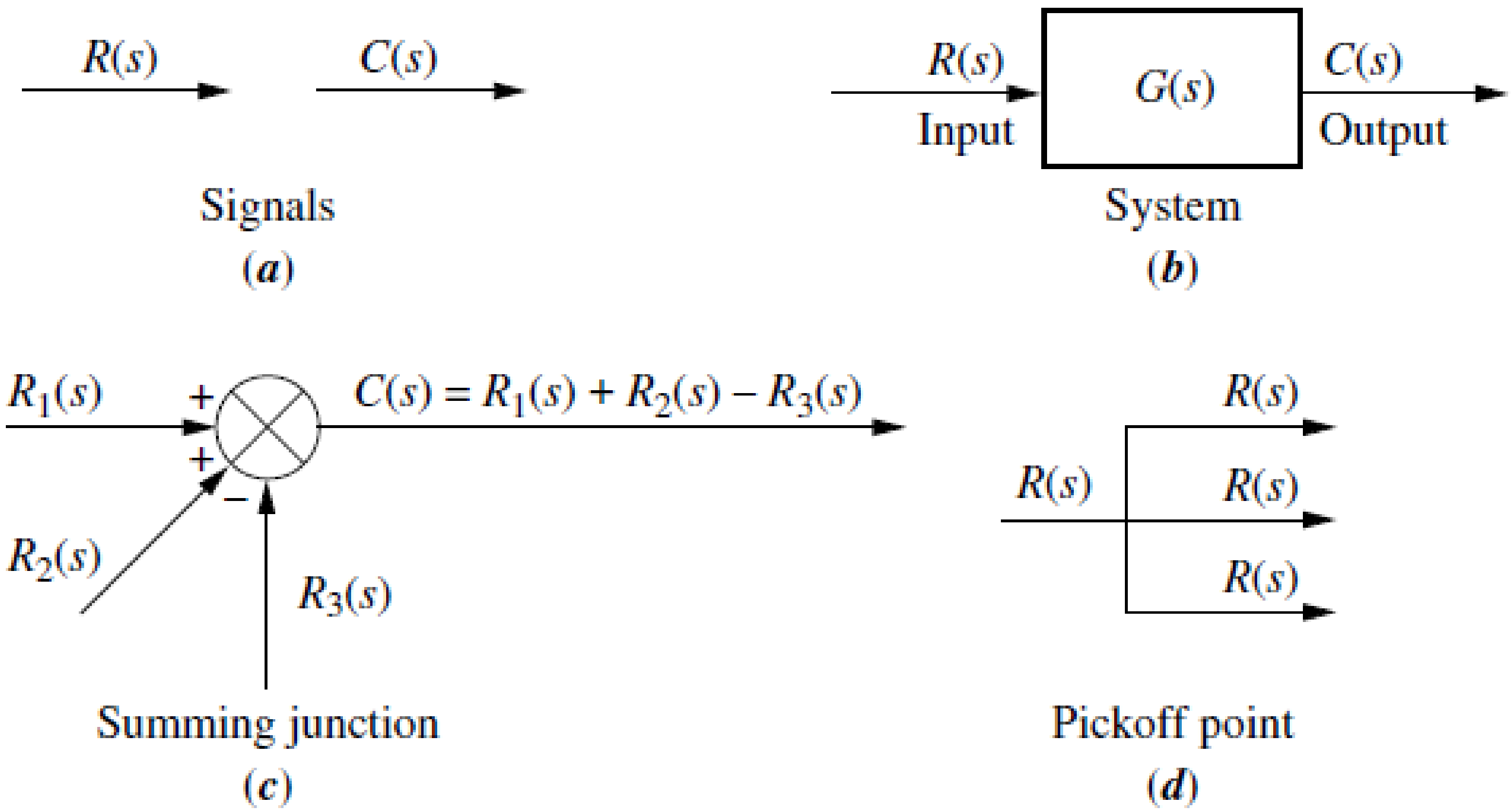


5.1.1 Components of a Block Diagram for Linear Time Invariant (LTI) Systems

Block diagram has four components:

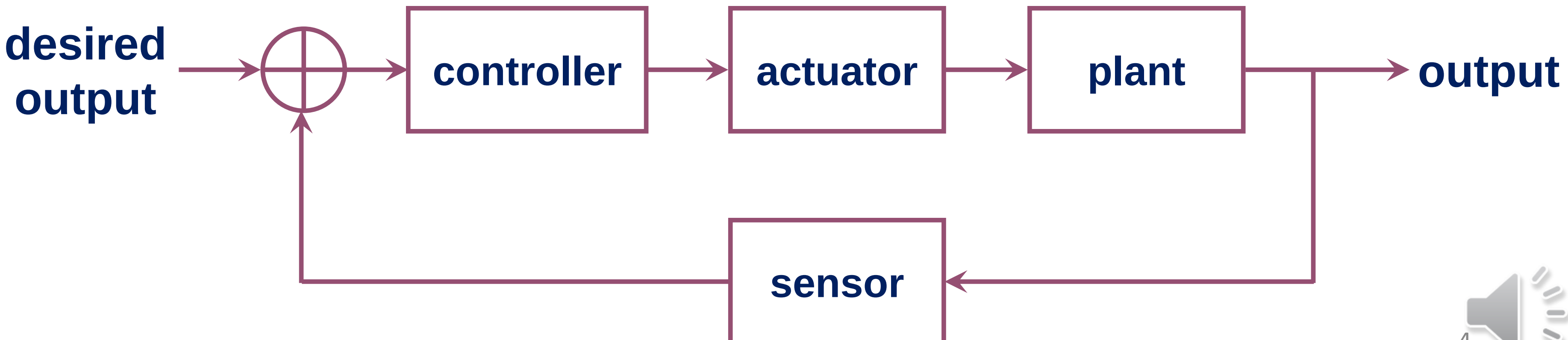
(1) Signals ***(2) System/ block***

(3) Summing junction ***(4) Pick-off/ Take-off point***



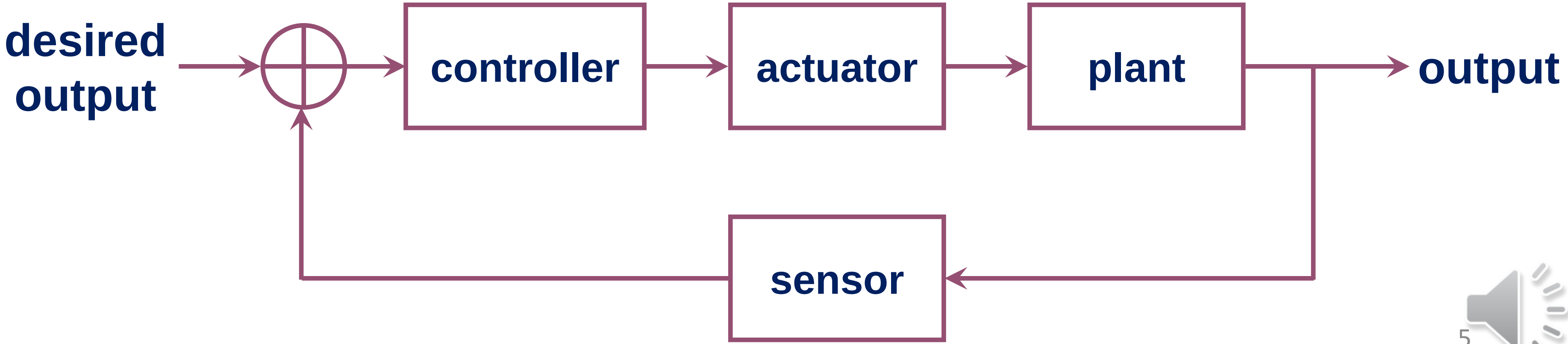
5.1.2 Generic Feedback Control System

- Input is the output we want the system to have.
- Summing junction subtracts the measured output from the desired output, difference is error signal.
- Controller acts based on magnitude of error signal.
- Actuator provides external power to system and effects changes based on controller output.
- Plant is the process we are trying to control.



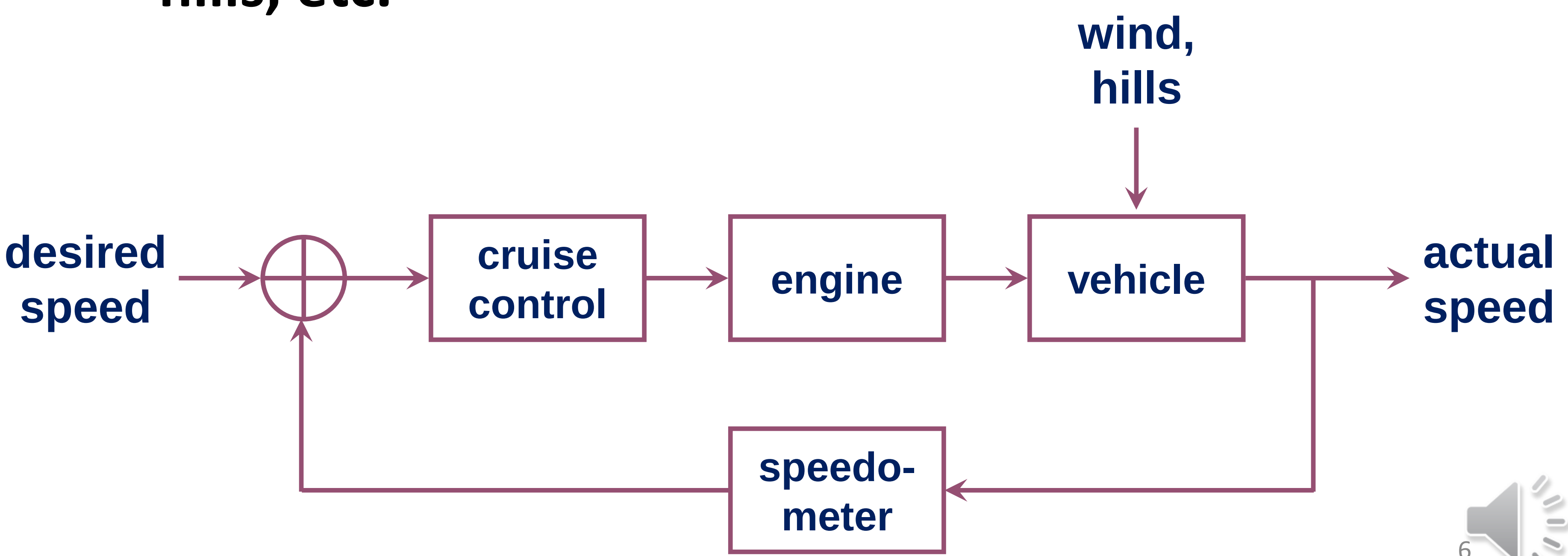
Generic Feedback Control System (Cont.)

- This is a general model, and may not be the same for every feedback control system
- Systems can have additional inputs known as disturbances into or between processes
- Can combine processes; typically controller and actuator are combined
- Describe and draw schematic, then recast your model into this form if possible



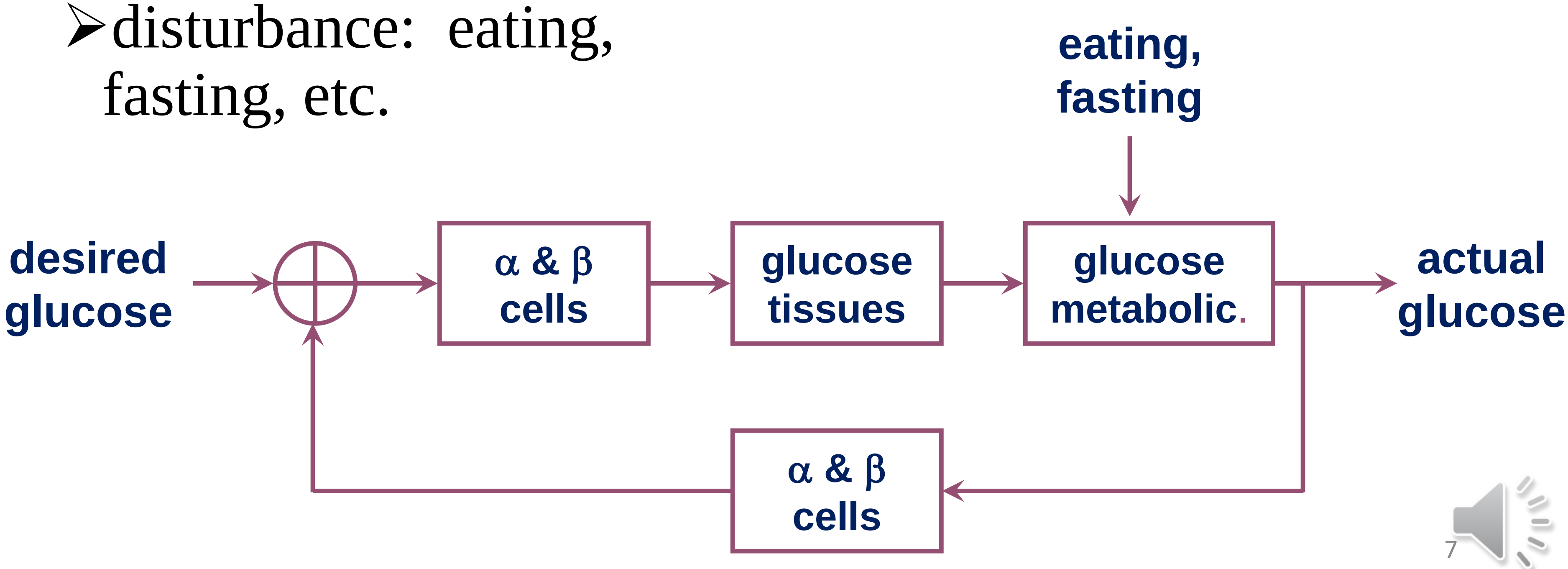
Example 1 :Cruise Control System Revisited

- input: desired speed
- output: actual speed
- error: desired speed minus measured speed
- disturbance: wind, hills, etc.
- controller: cruise control unit
- actuator: engine
- plant: vehicle dynamics
- sensor: speedometer



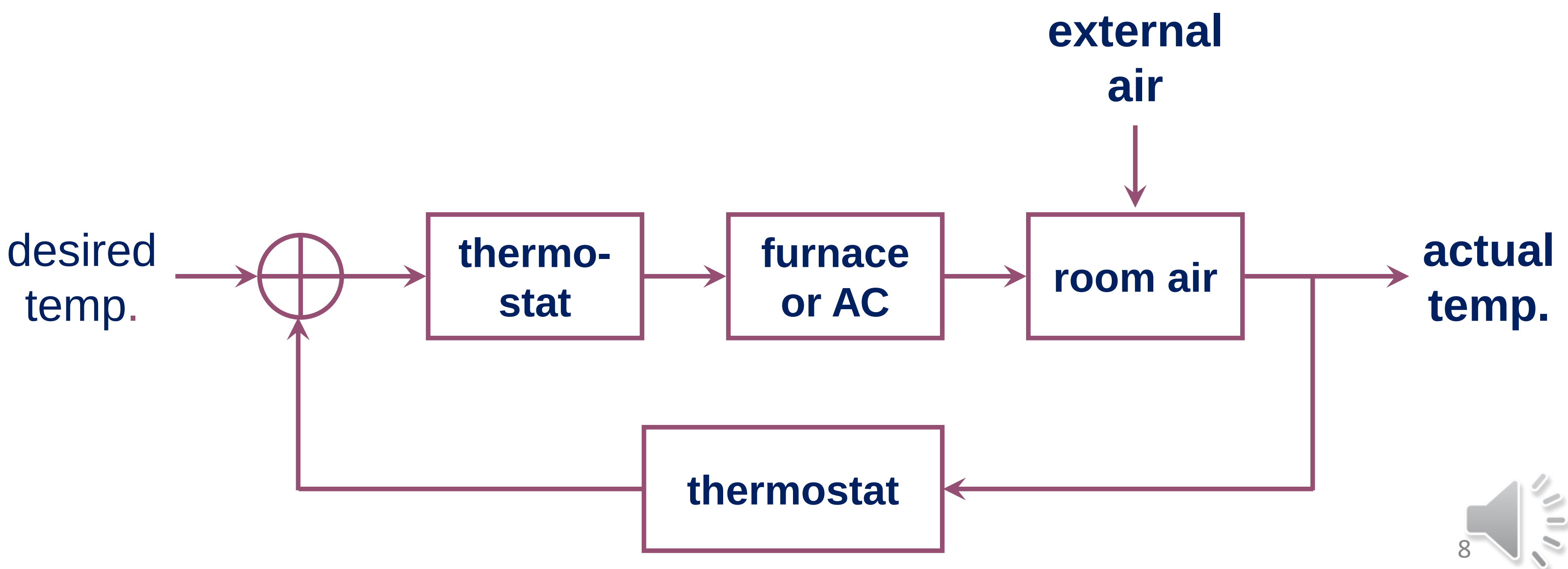
Example 2 : Glucose Regulation Revisited

- input: desired blood glucose
- output: actual blood glucose
- error: desired minus measured blood glucose
- disturbance: eating, fasting, etc.
- controller: α and β cells
- actuator: glucose storing or releasing tissues
- plant: glucose metabolism
- sensor: α and β cells (again)



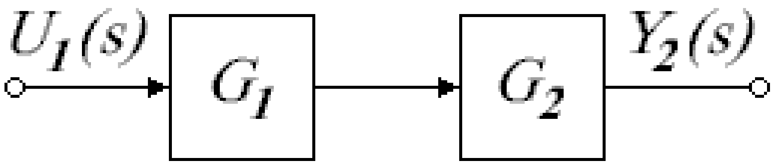
Example 3 :Thermostat Example

- ❖Set thermostat to desired room temperature.
- ❖Thermostat measures room temperature.
- ❖Furnace or AC turn on if the measured temperature is < or > the desired temperature.
- ❖Air from furnace or AC changes room air temperature.



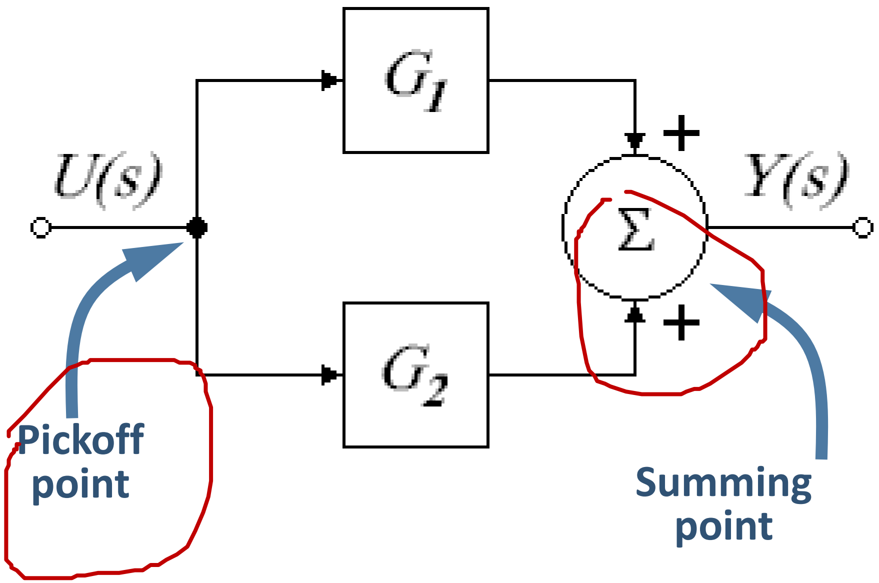
§ 5.2 Elementary Block Diagrams

Blocks in cascade



$$\frac{Y_2(s)}{U_1(s)} = G_2 G_1$$

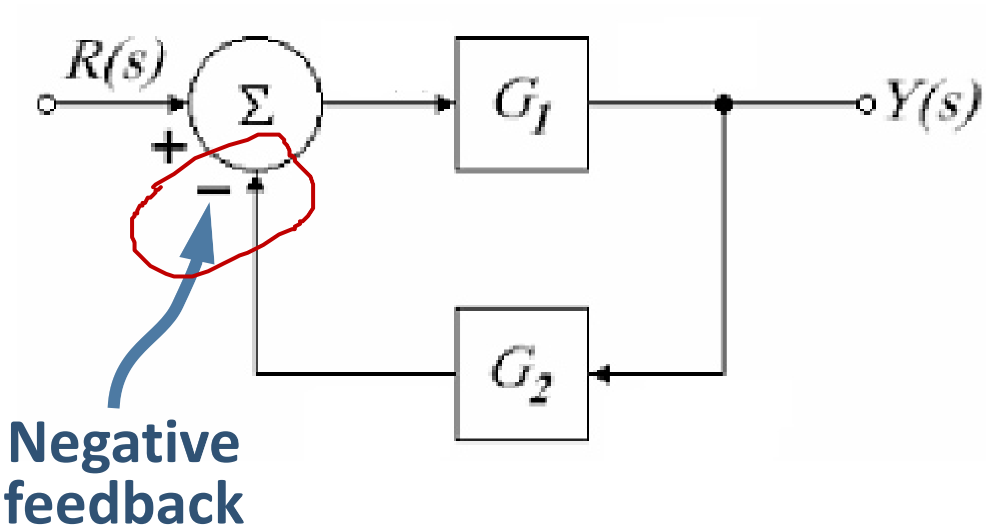
Blocks in parallel with their outputs added



$$\frac{Y(s)}{U(s)} = G_1 + G_2$$

Elementary Block Diagrams (Cont.)

Single loop negative feedback



$$\frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1G_2}$$

$$Y(s) = \{R(s) - Y(s)\cdot G_2\}\cdot G_1$$

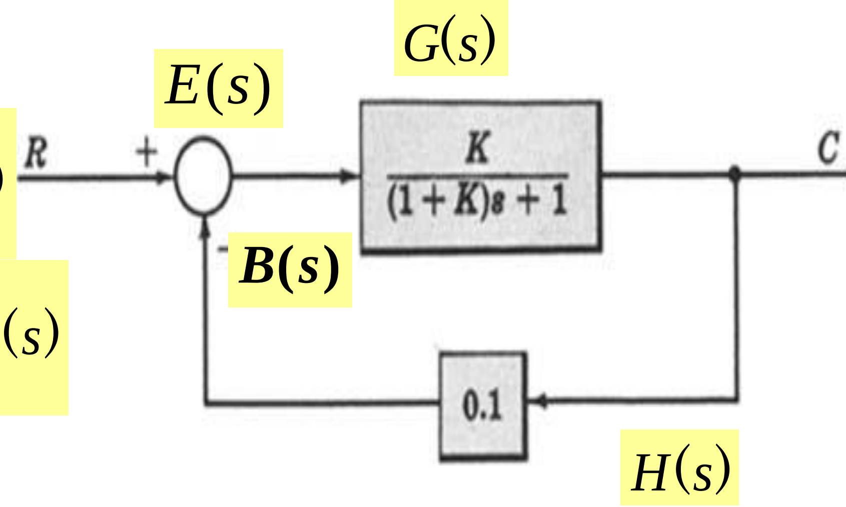
$$Y(s) = G_1R(s) - G_1G_2Y(s)$$

$$Y(s)\{1+G_1G_2\} = G_1R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1G_2}$$

What about single loop with positive feedback?

Definitions



1. **Open loop transfer function** $\frac{B(s)}{E(s)} = G(s)H(s)$

2. **Feed Forward Transfer function** $\frac{C(s)}{E(s)} = G(s)$

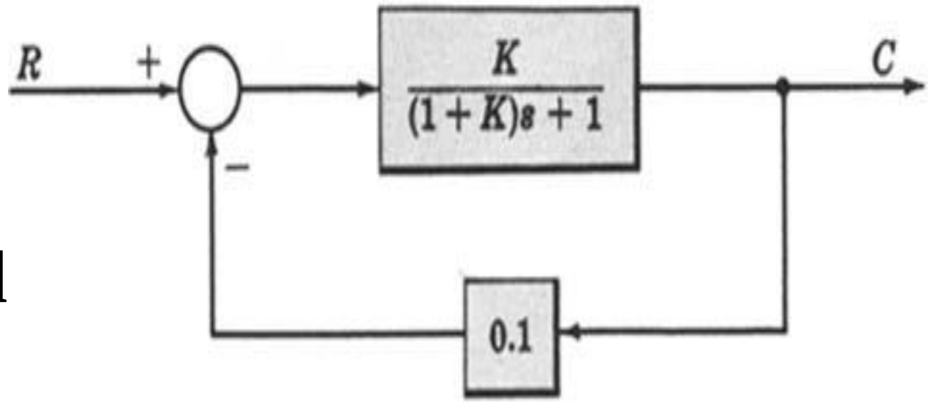
3. **feedback ratio** $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$

4. **error ratio** $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$

5. **closed loop transfer function (control ratio)** $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

6. **characteristic equation** $1+G(s)H(s) = 0$

Characteristic Equation



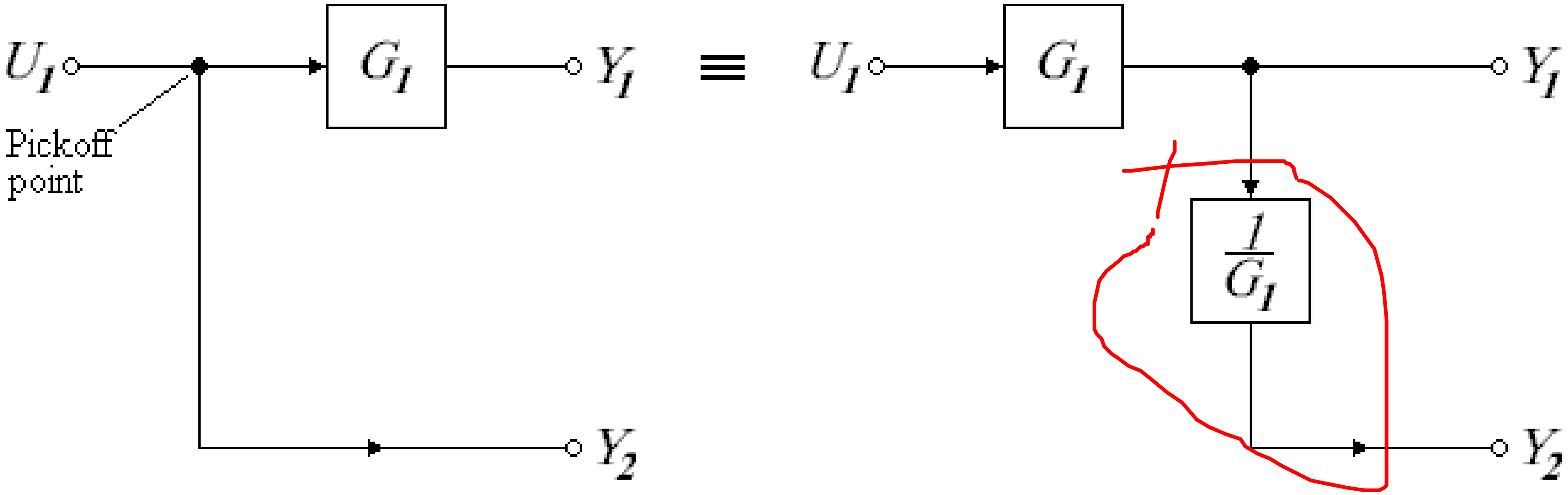
- The control ratio is the closed loop transfer function of the system.

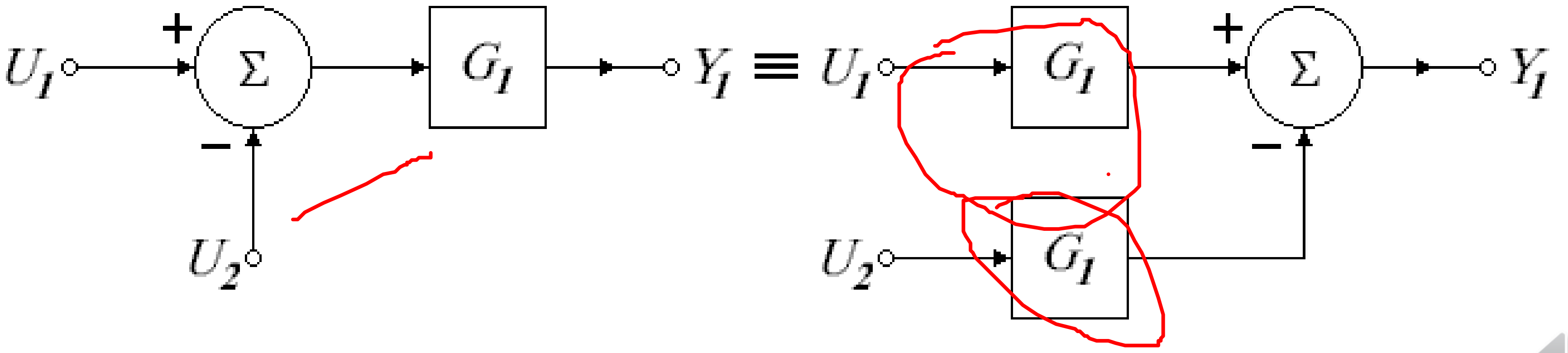
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

- The denominator of closed loop transfer function determines the characteristic equation of the system.
- Which is usually determined as:

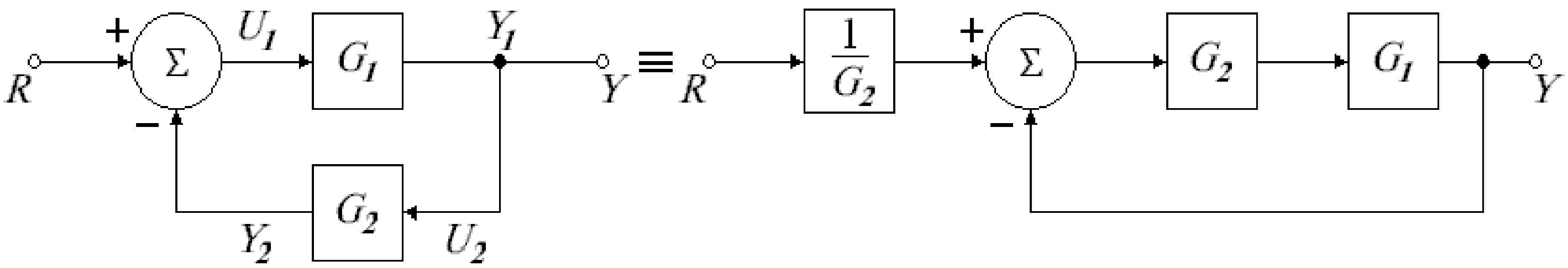
$$1 \pm G(s)H(s) = 0$$

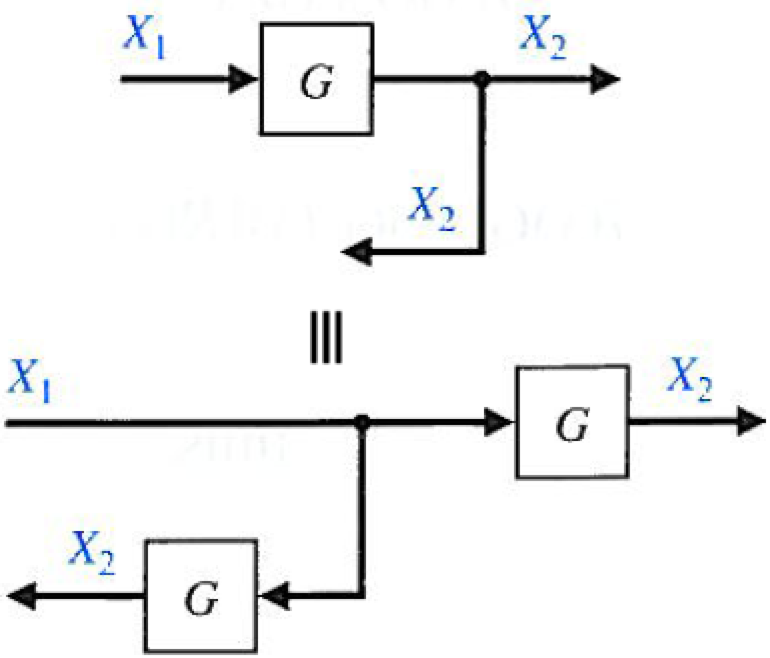
§ 5.3 Block Diagram Algebra

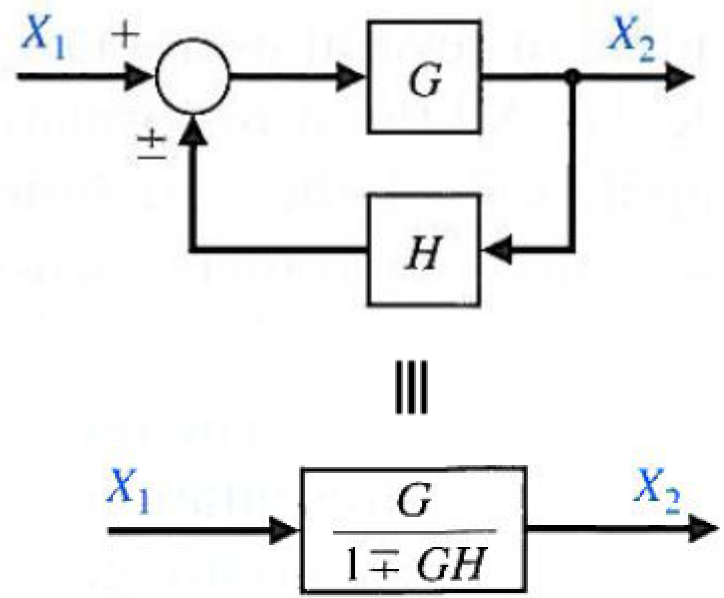




Block Diagram Algebra (Cont.)

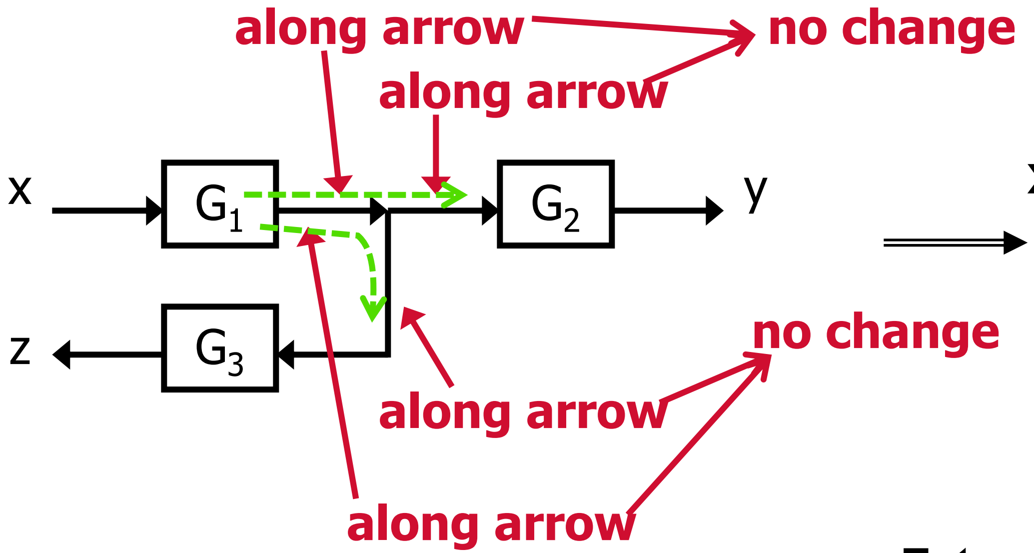


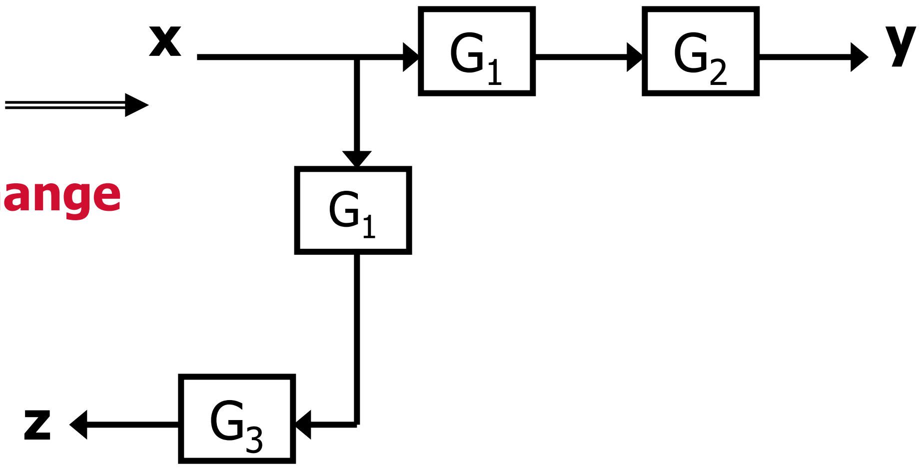




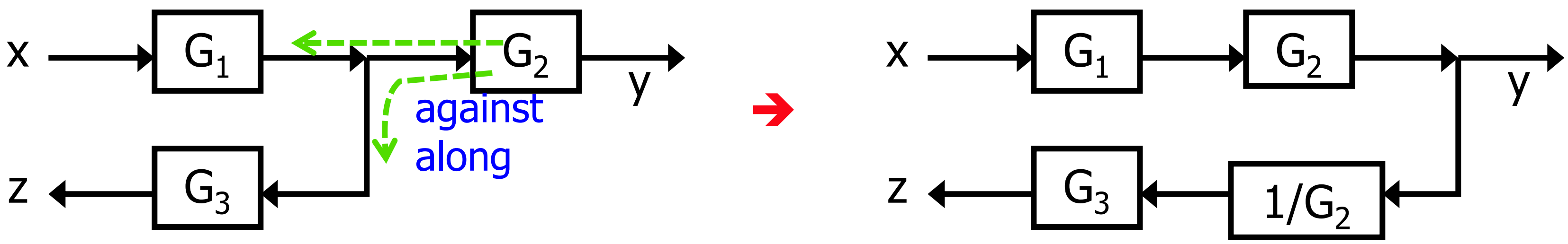
- Move a block (G1) across a pick-up point / summation into all touching lines:

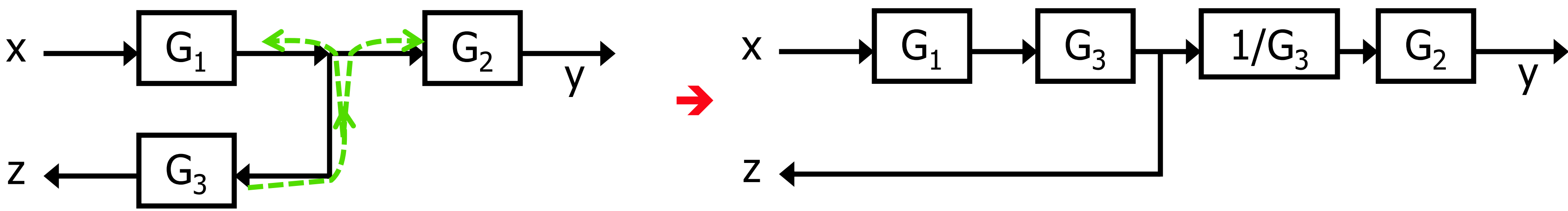
- **For example:**



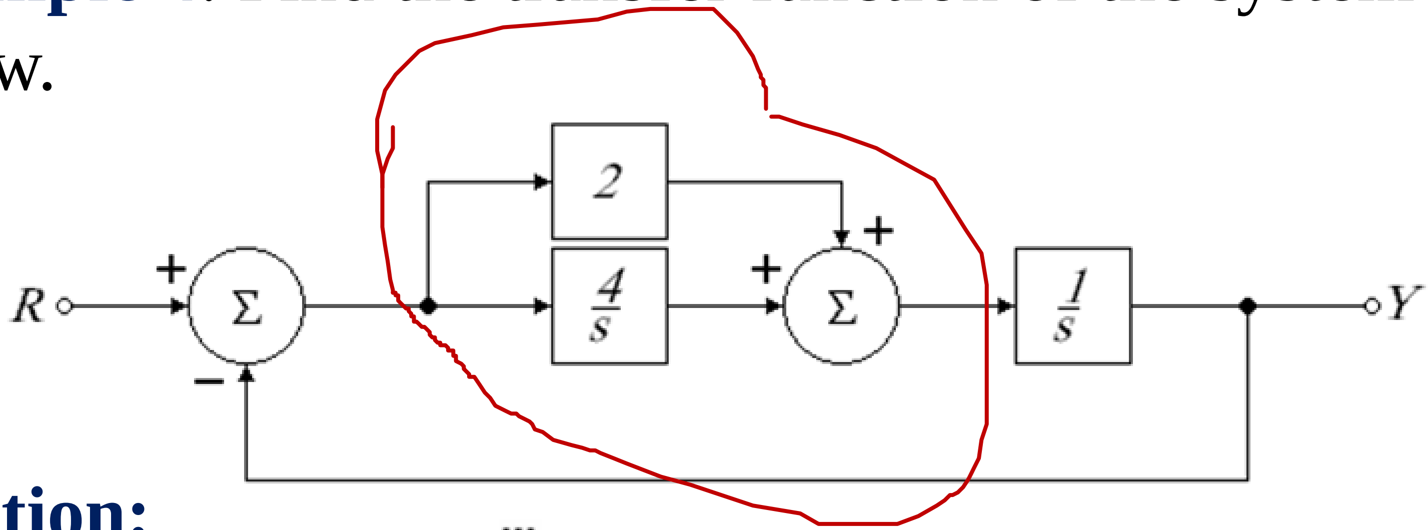


against, against



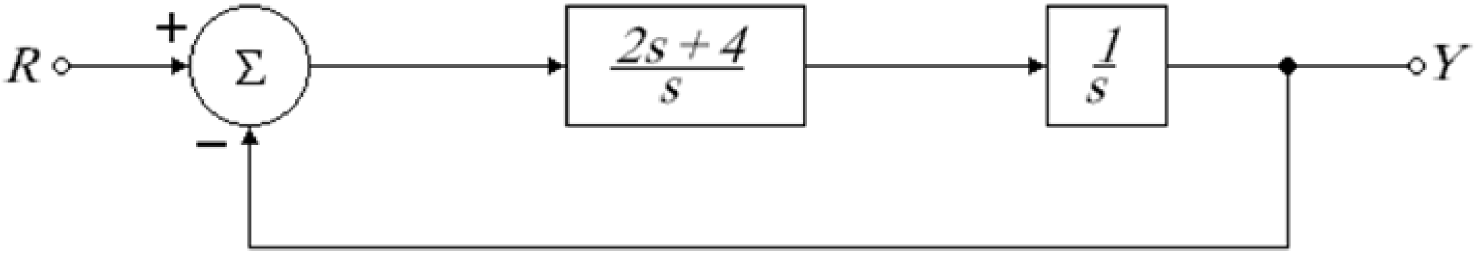


Example 4: Find the transfer function of the system shown below.



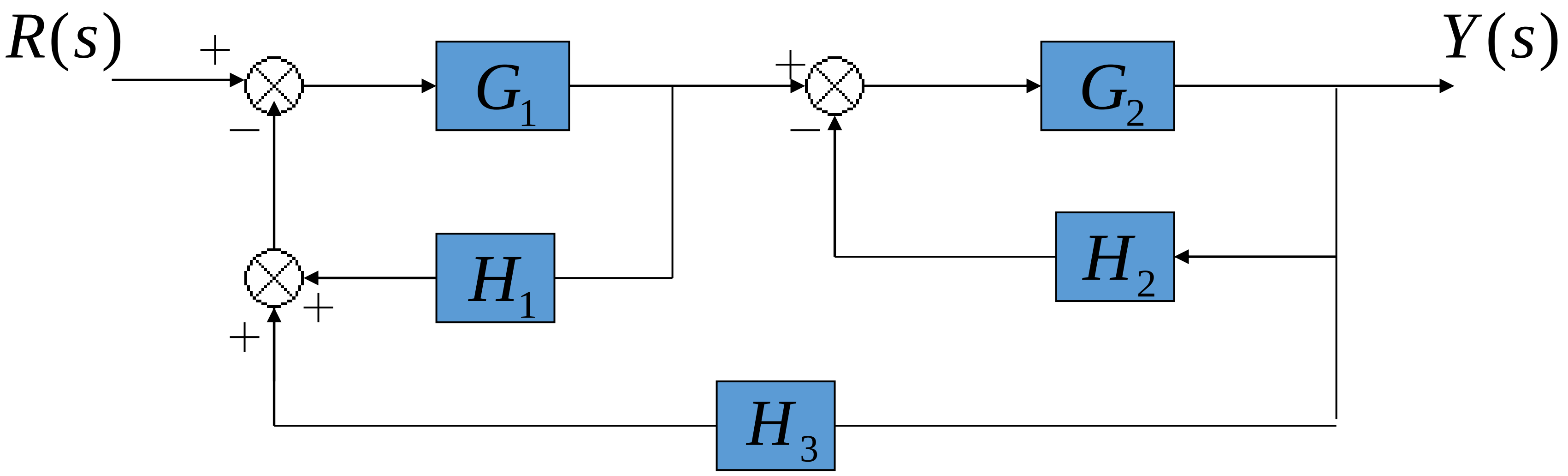
Solution:

≡

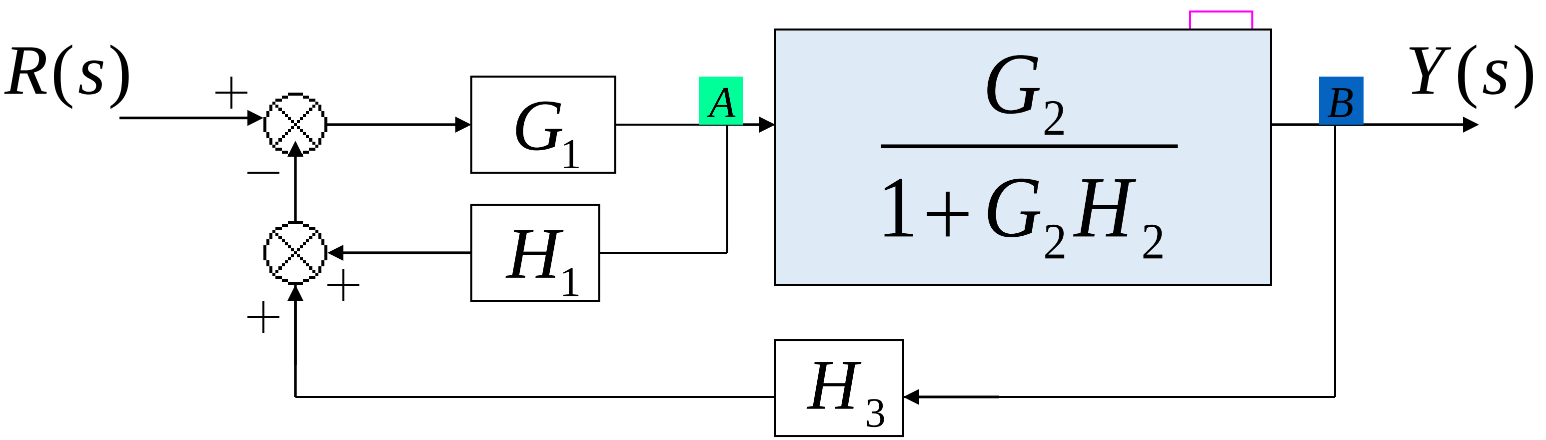


$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{2s+4}{s^2}}{1+\frac{2s+4}{s^2}} = \underline{\underline{\frac{2s+4}{s^2+2s+4}}}$$

Example 5: Find the transfer function of the following block diagrams

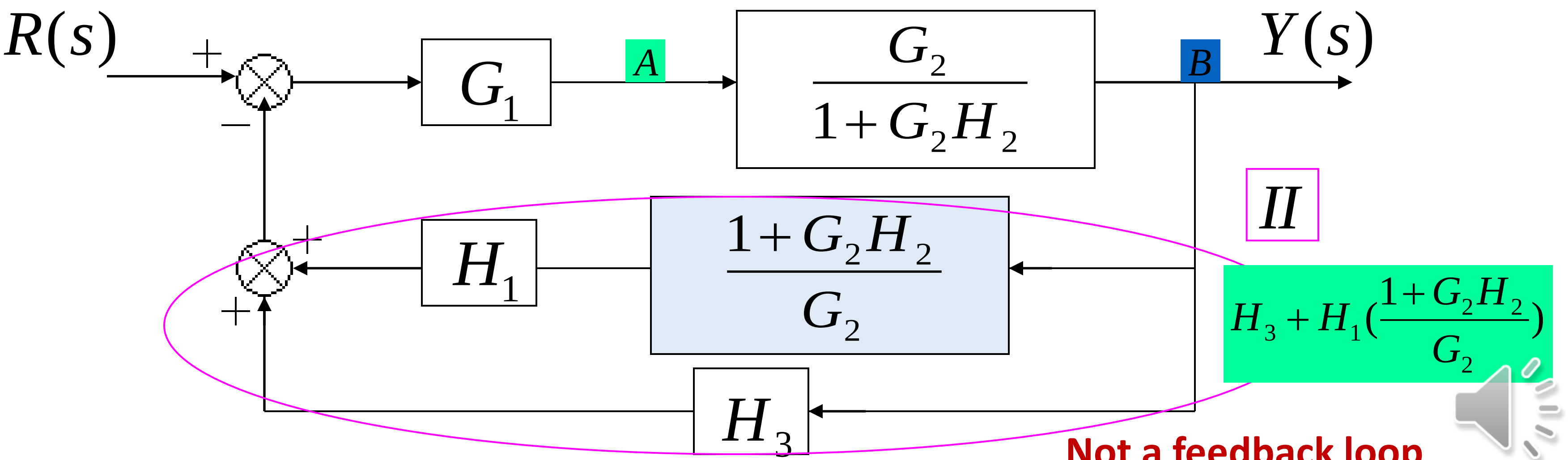


1. Eliminate loop I

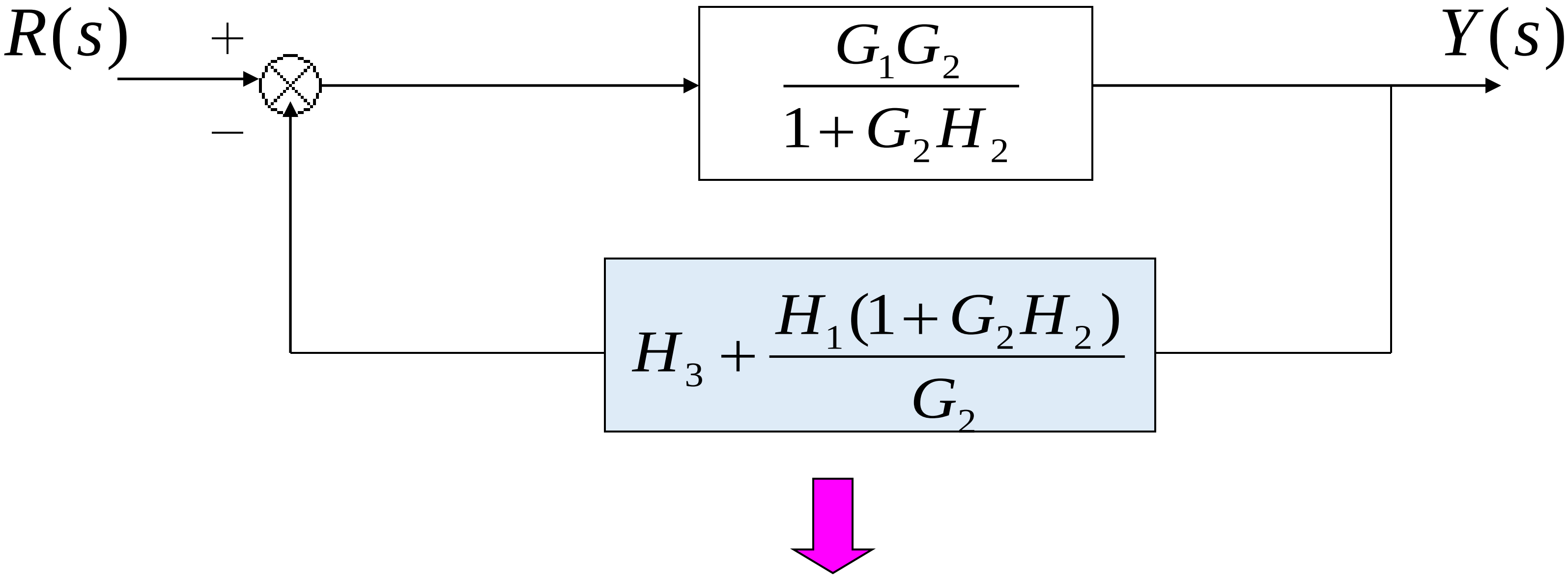


2. Moving pickoff point A behind block

$$\frac{G_2}{1+G_2H_2}$$

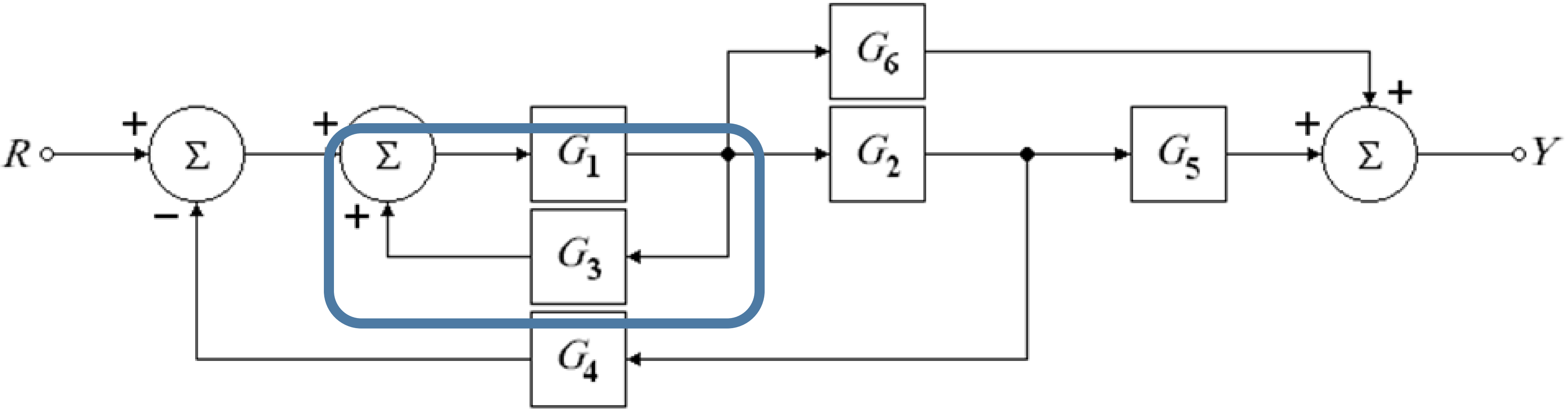


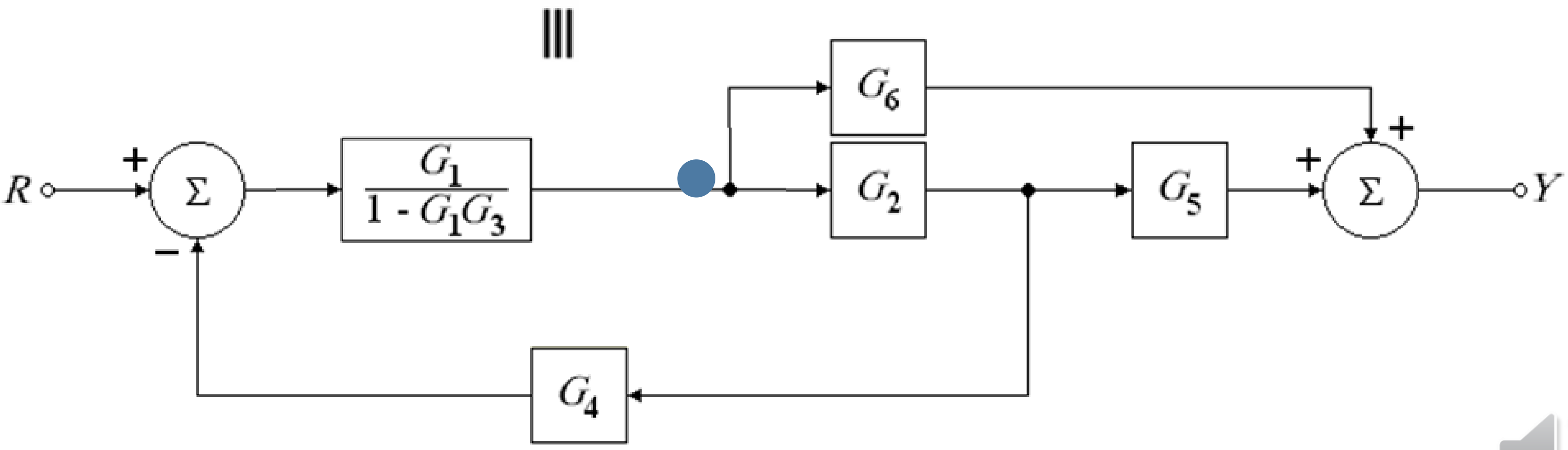
3. Eliminate loop II



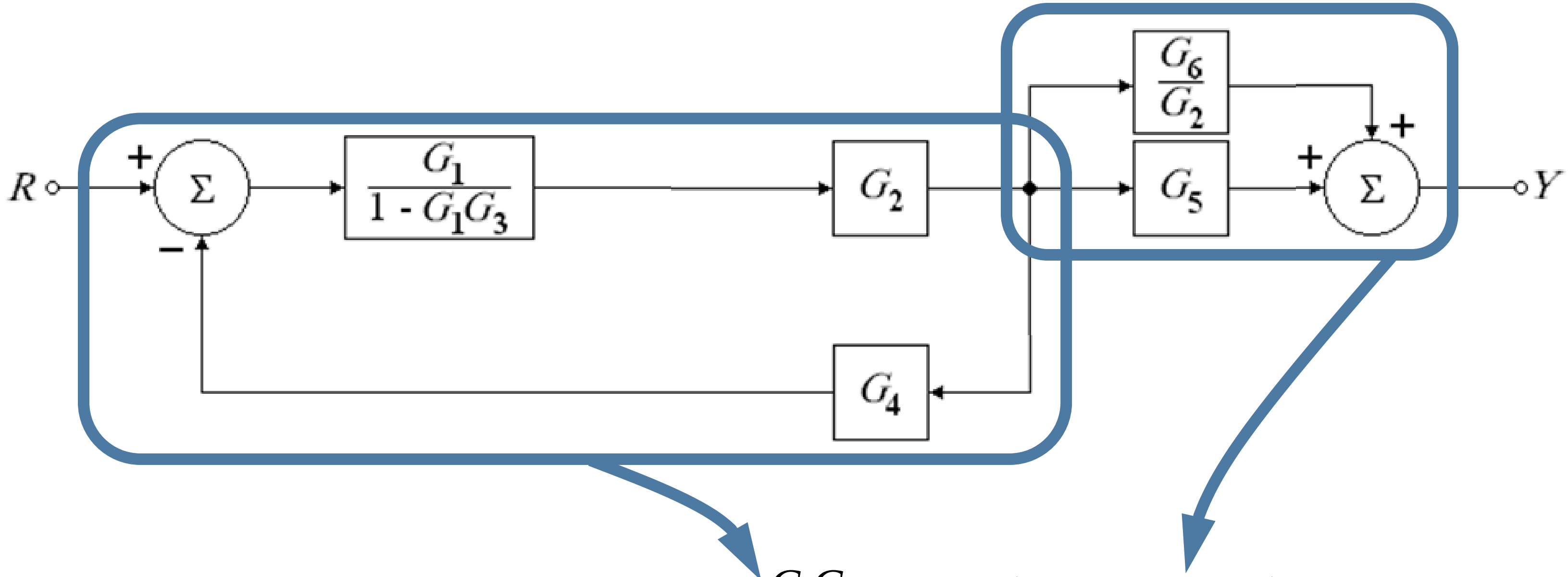
$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_3 + G_1 H_1 + G_1 G_2 H_1 H_2}$$

Example6: Find the transfer function of the system shown below.





Example 6 (Cont.) :

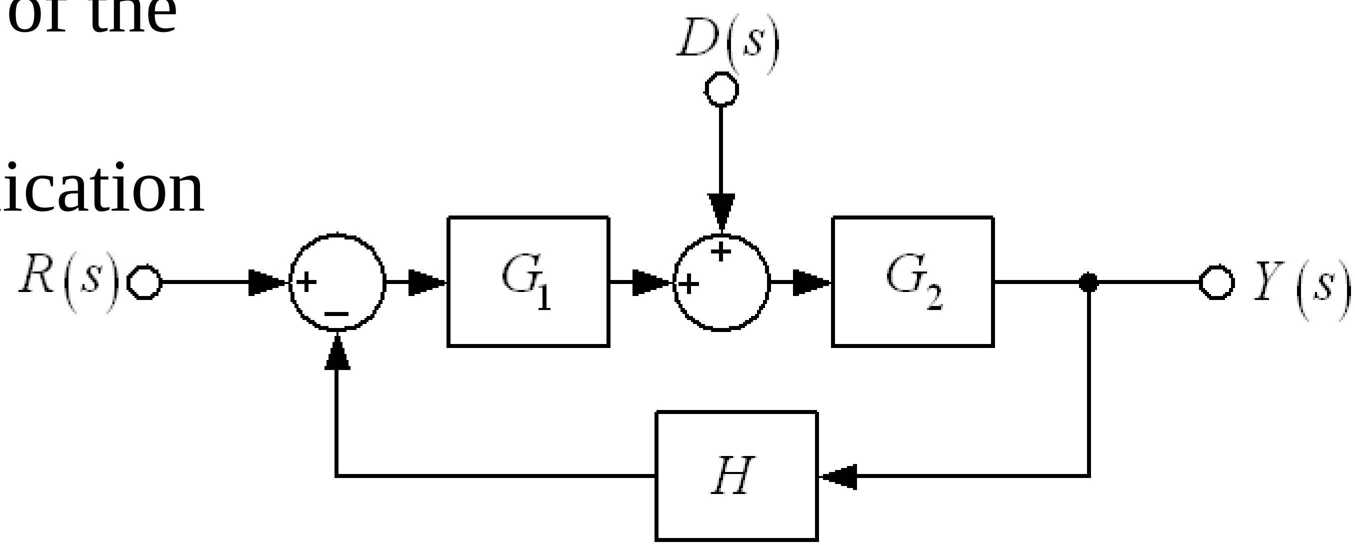


$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{G_1G_2}{1-G_1G_3}}{1+\frac{G_1G_2G_4}{1-G_1G_3}} \cdot \left(\frac{G_6}{G_2} + G_5 \right)$$

$$= \frac{G_1G_2G_5 + G_1G_6}{1 - G_1G_3 + G_1G_2G_4}$$

Example 7:

Find the response of the system $Y(s)$ to simultaneous application of the reference input $R(s)$ and disturbance $D(s)$.



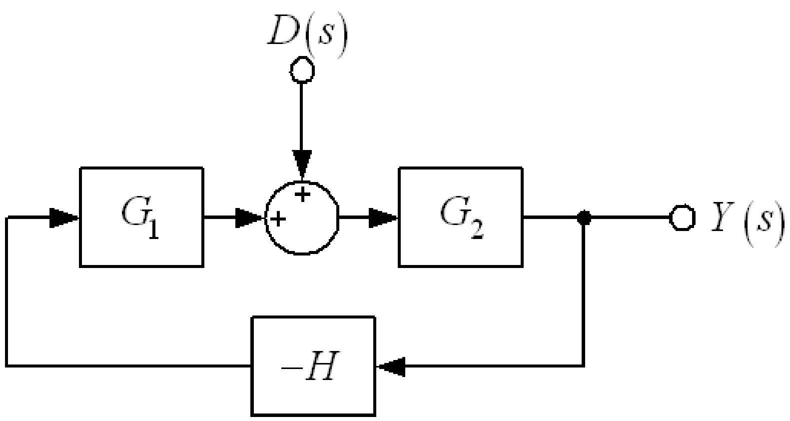
Solution:

When $D(s) = 0$,

$$\frac{Y_R(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H}$$

When $R(s) = 0$,

$$\frac{Y_D(s)}{D(s)} = \frac{G_2}{1 - (-G_1) G_2 H} = \frac{G_2}{1 + G_1 G_2 H}$$



Example 7 (Cont.):
Note: LTI system

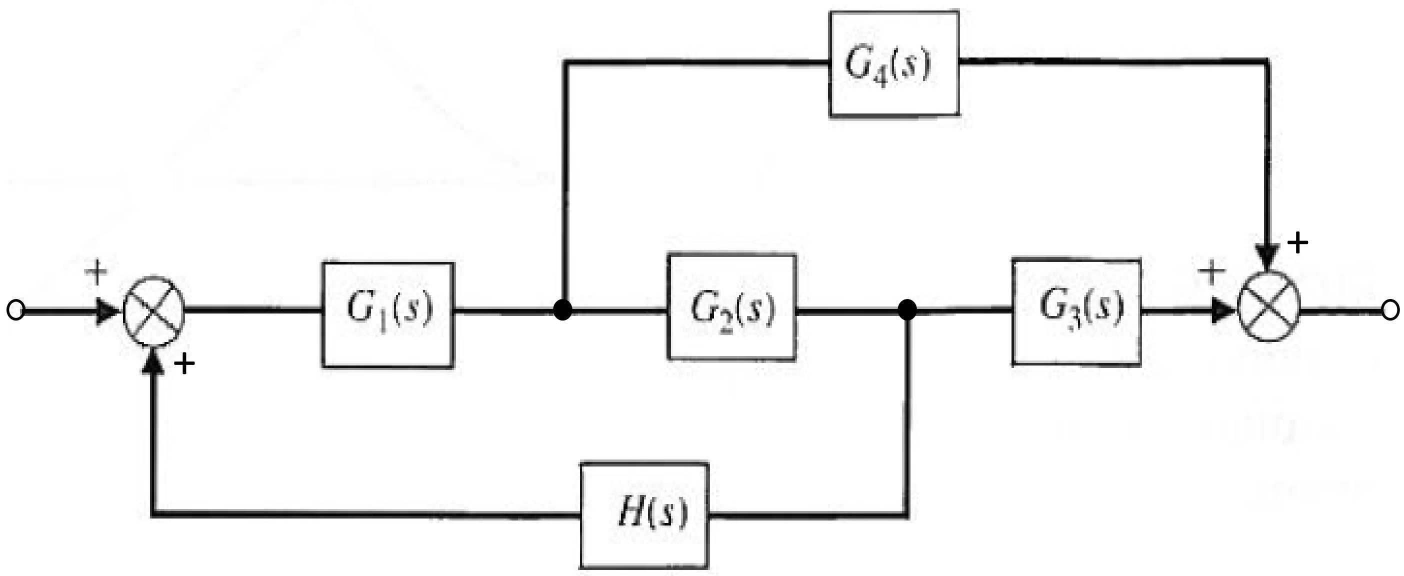
$$Y(s) = Y_R(s) + Y_D(s)$$

$$Y(s) = \frac{G_1G_2}{1+G_1G_2H}R(s) + \frac{G_2}{1+G_1G_2H}D(s)$$

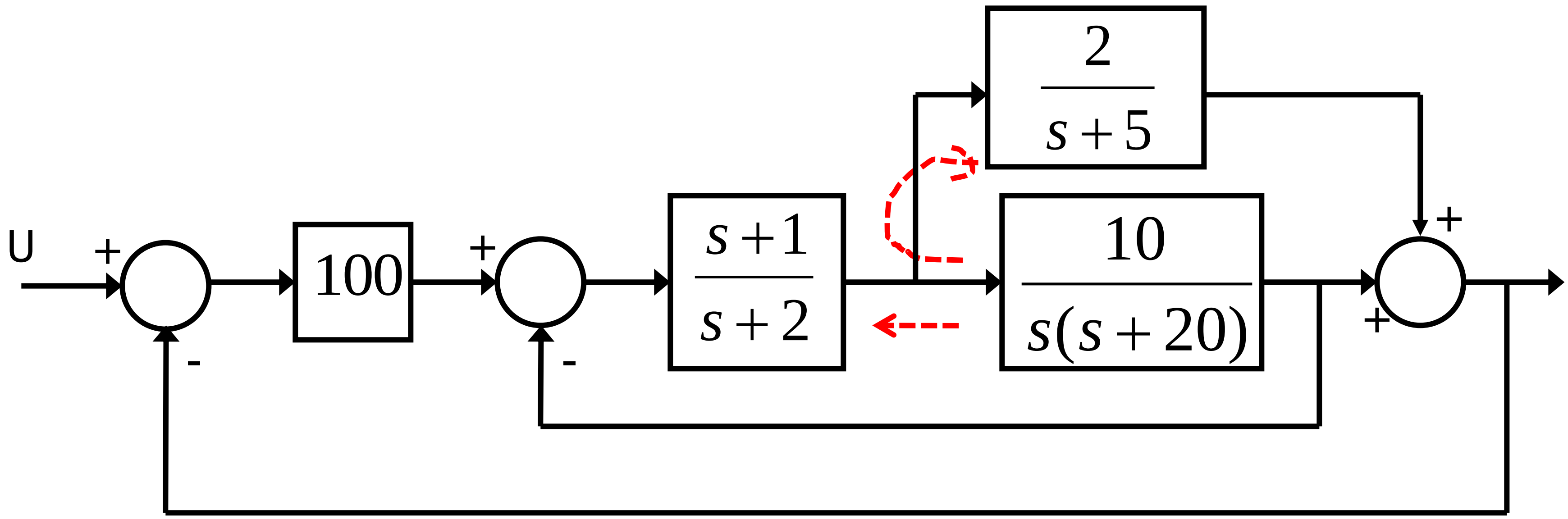
$$Y(s) = \frac{G_2}{1+G_1G_2H}\left(G_1R(s) + D(s)\right)$$

Quiz
Example 8:

Find the overall transfer function of the system given below.



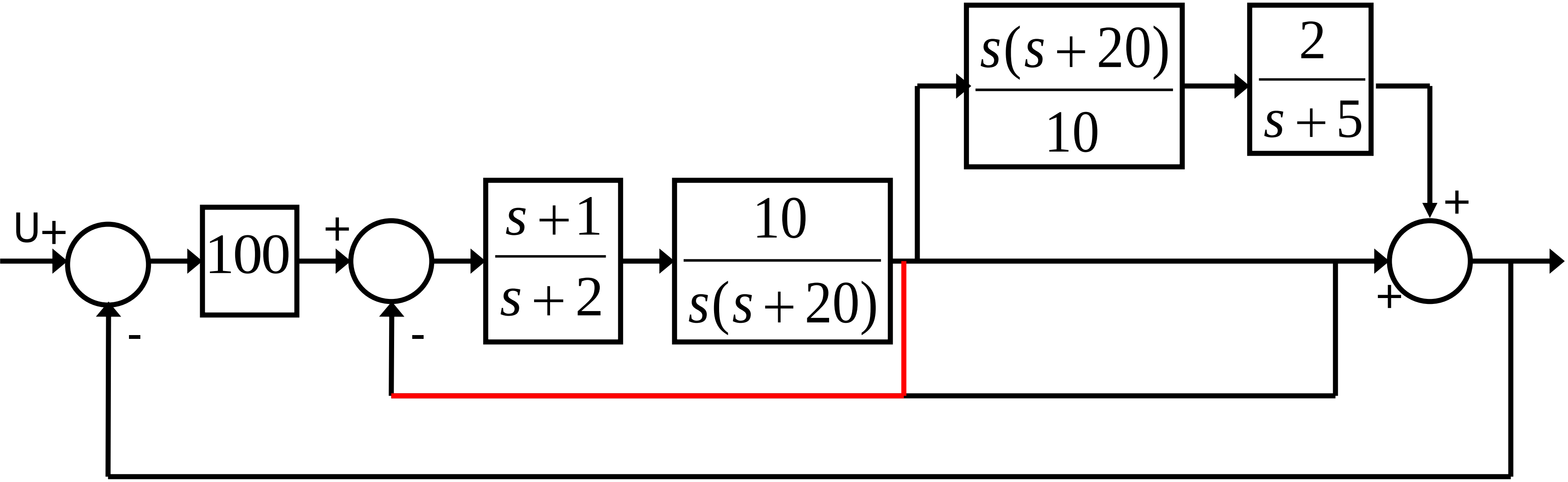
Example 9: Find the TF from U to Y:

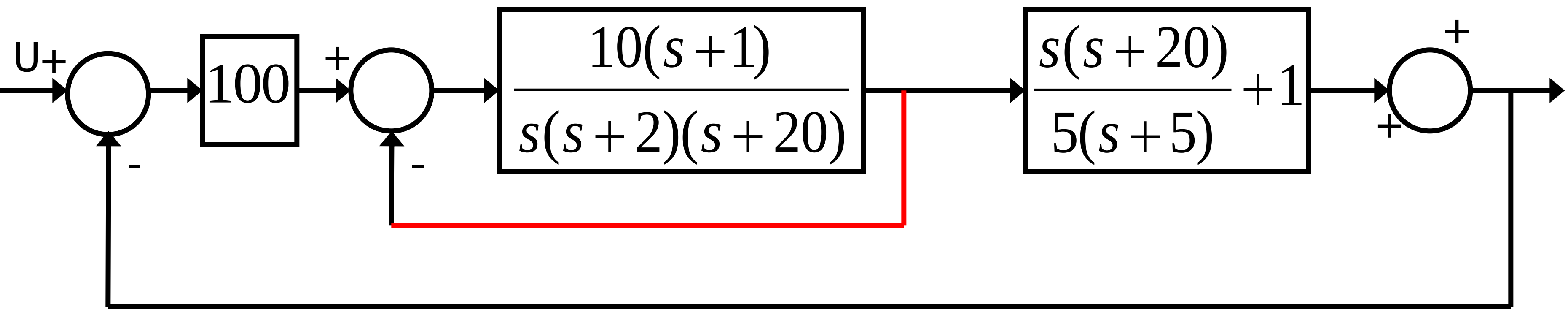


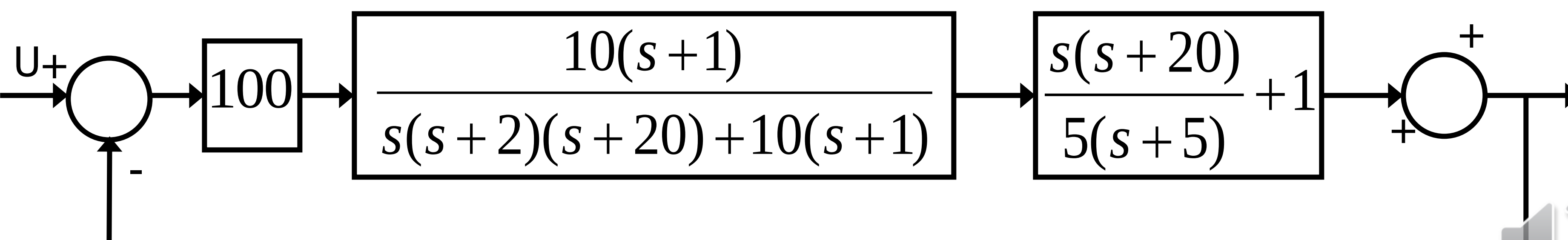
- No pure cascade/parallel/feedback
- Needs to move a block, but which one?

  Key: move one block to create pure cascade or parallel or feedback!

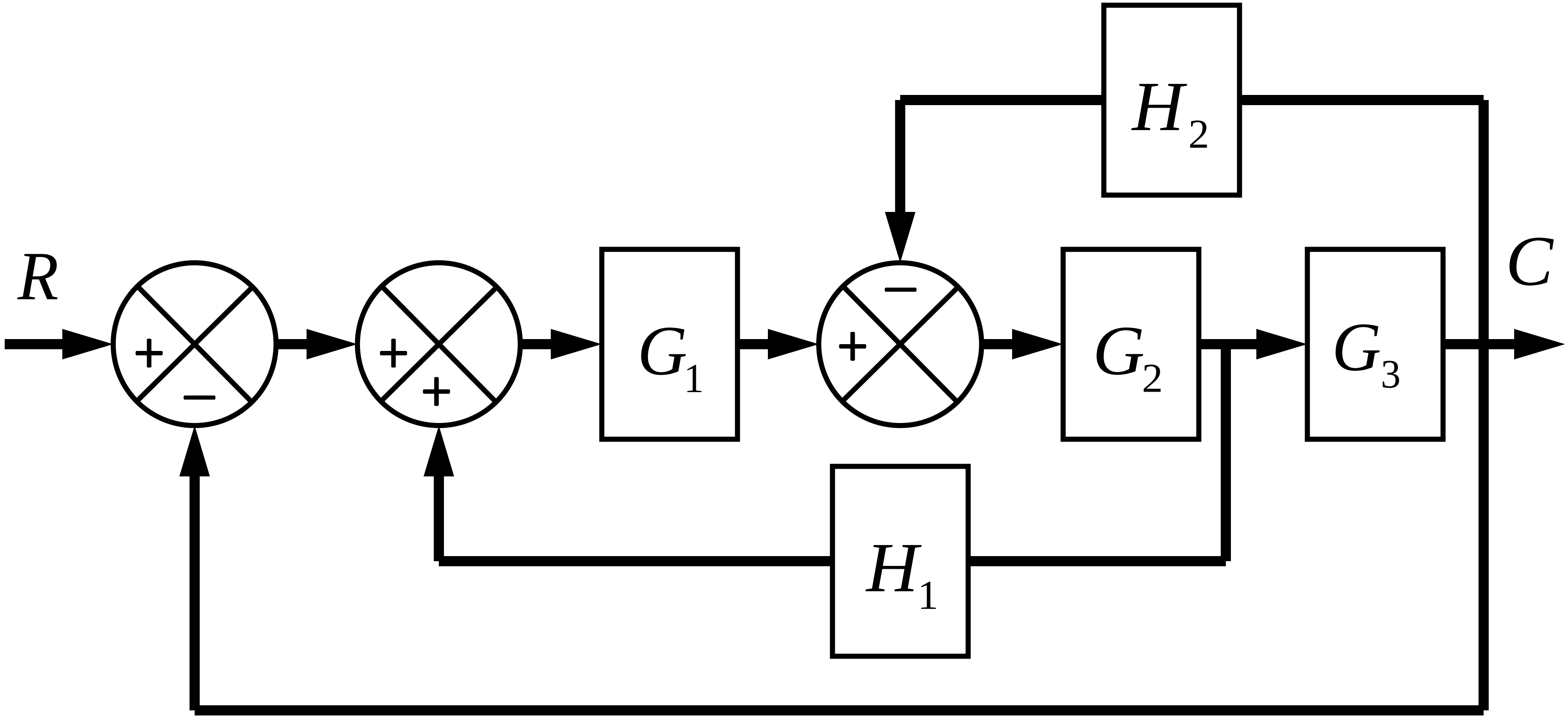
  So move $\frac{10}{s(s+20)}$ either left or right.


U
+
-
100
+
-
$\frac{s+1}{s+2}$
$\frac{10}{s(s+20)}$
$\frac{s(s+20)}{10}$
$\frac{2}{s+5}$
+
+
Y

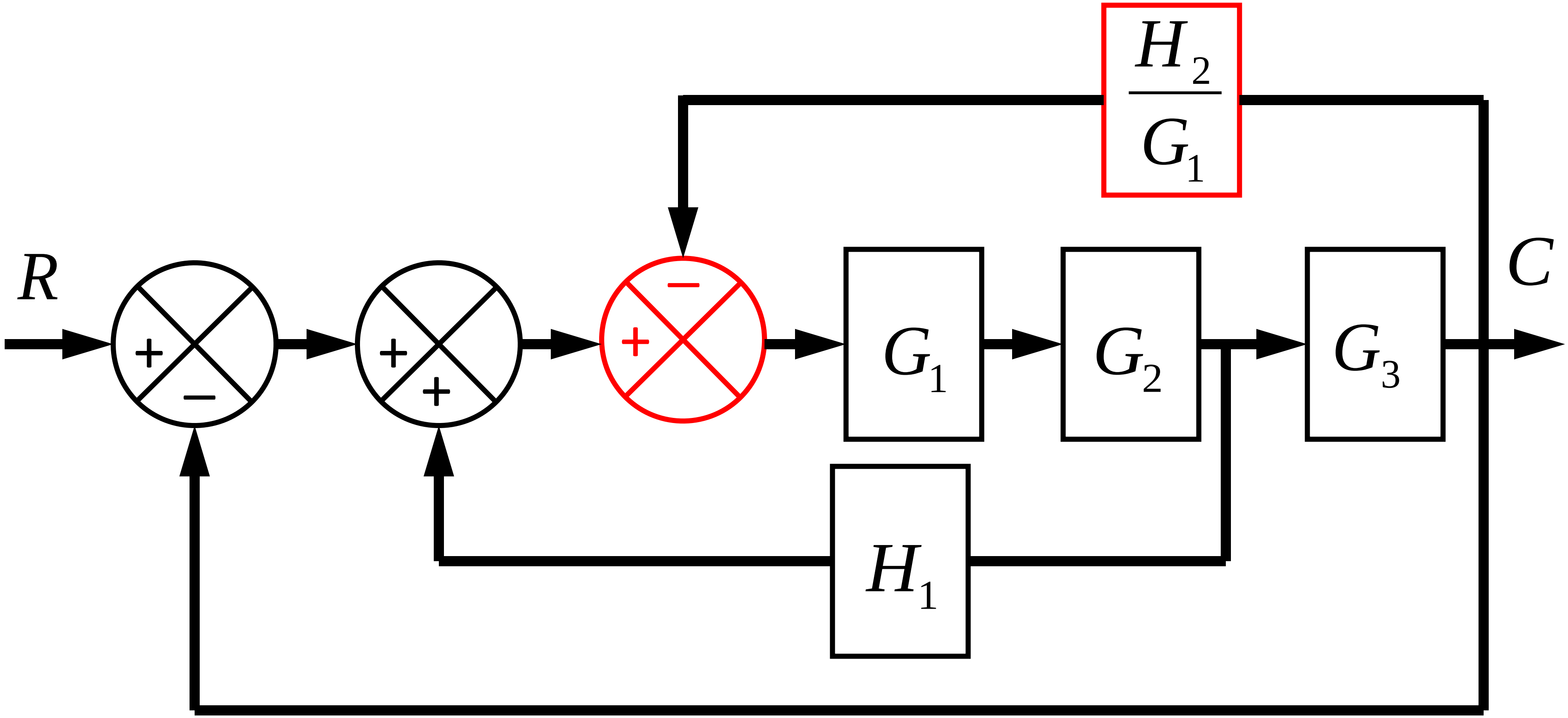

U
+
-
100
+
-
$\frac{10(s+1)}{s(s+2)(s+20)}$
$\frac{s(s+20)}{5(s+5)}+1$
+
+
Y


U
+
-
100
$\frac{10(s+1)}{s(s+2)(s+20)+10(s+1)}$
$\frac{s(s+20)}{5(s+5)}+1$
+
+
Y

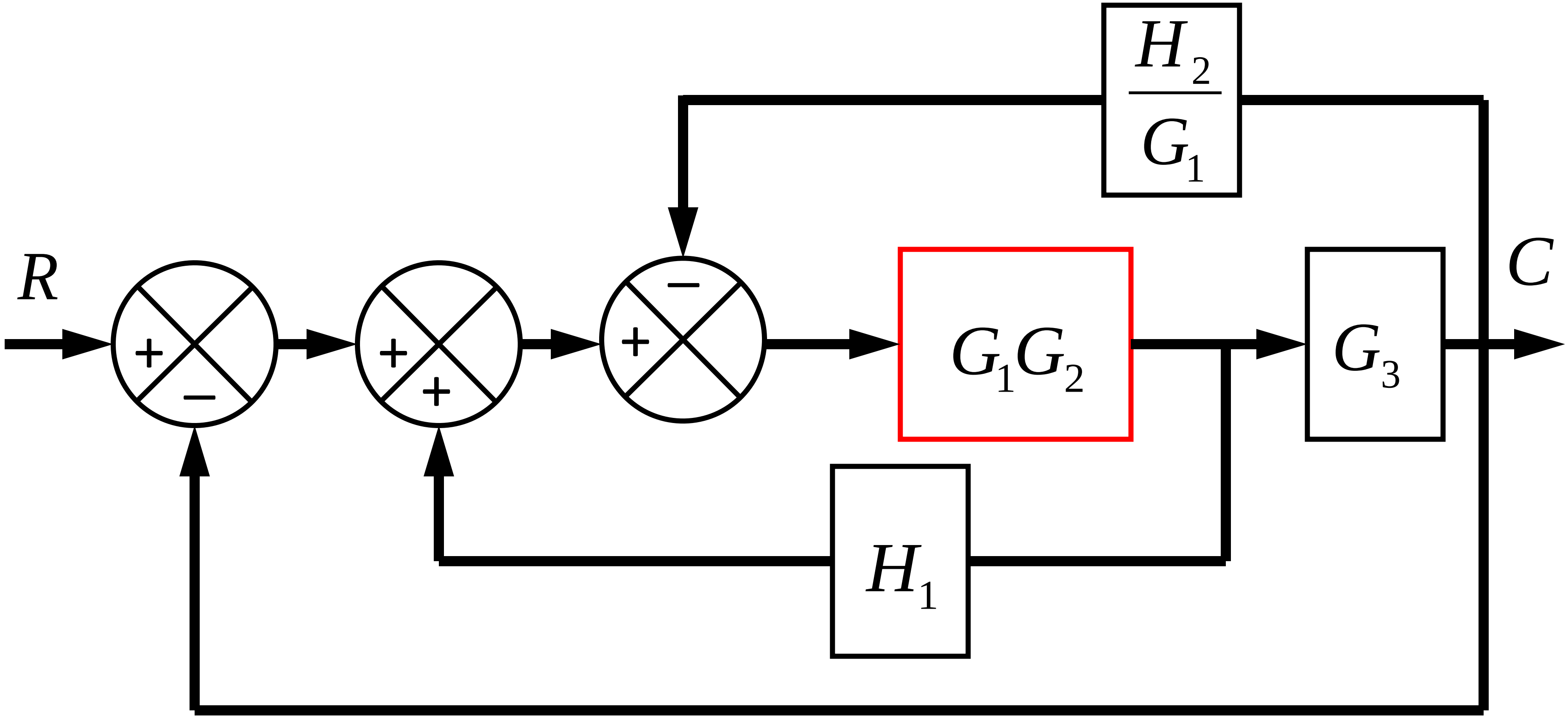
Example 10 : Find TF from R to C



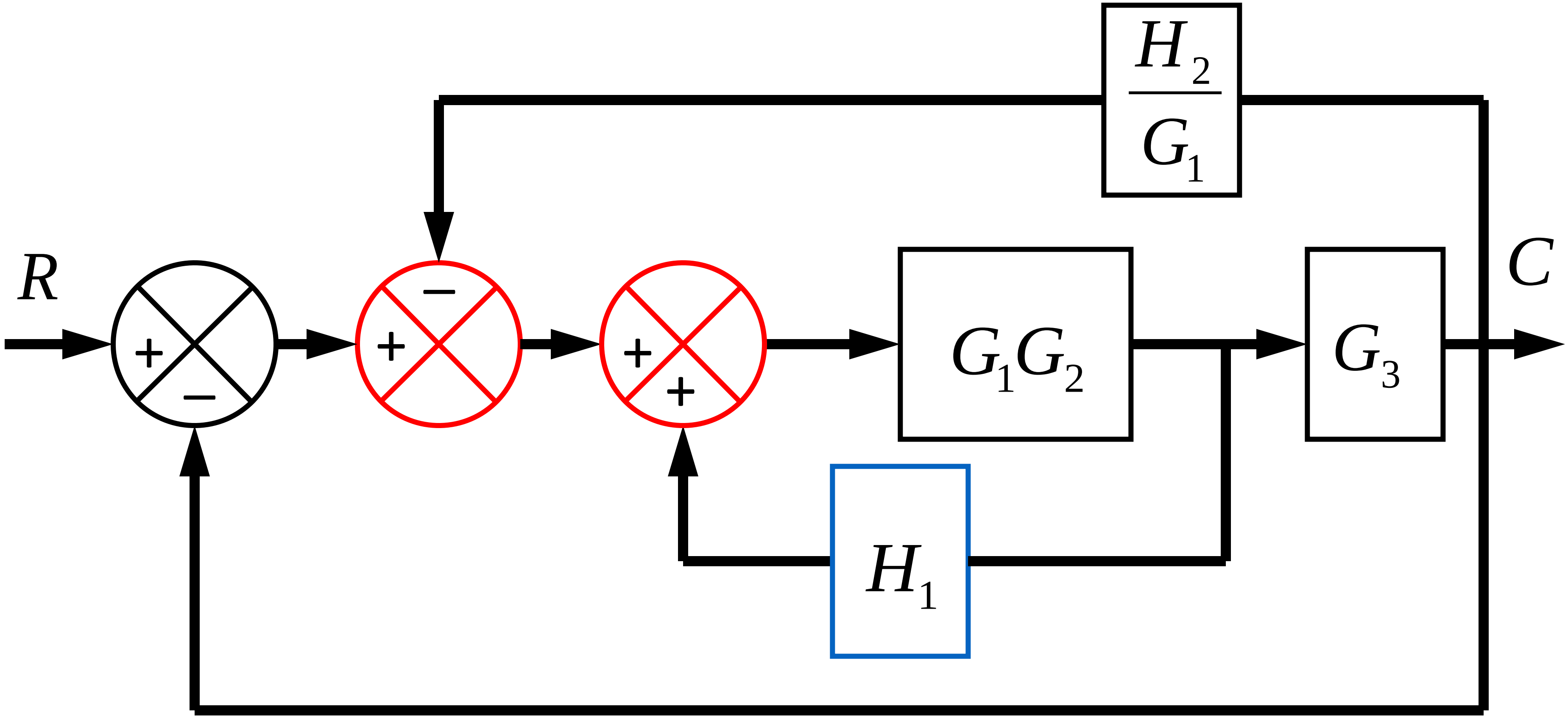
Example 10 : Cont.



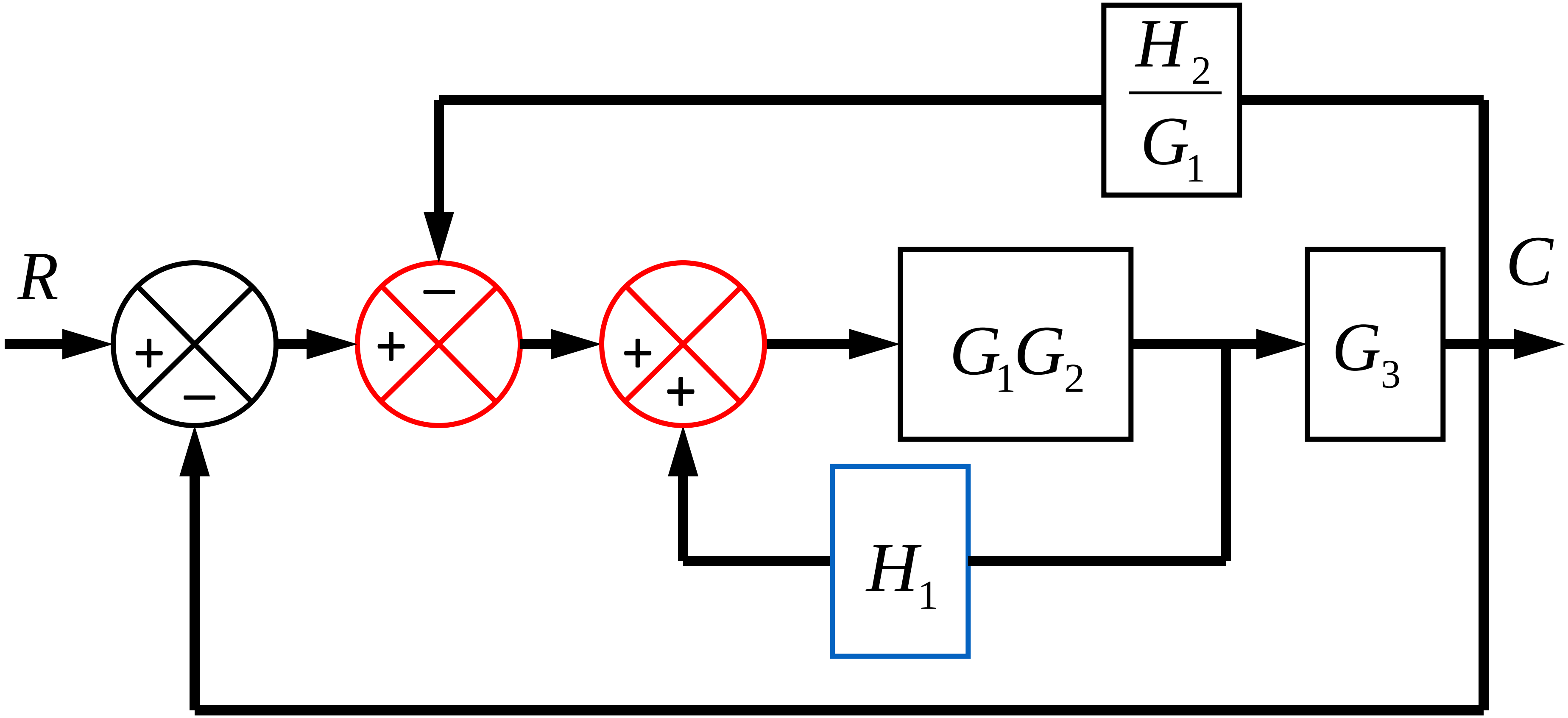
Example 10: Cont.



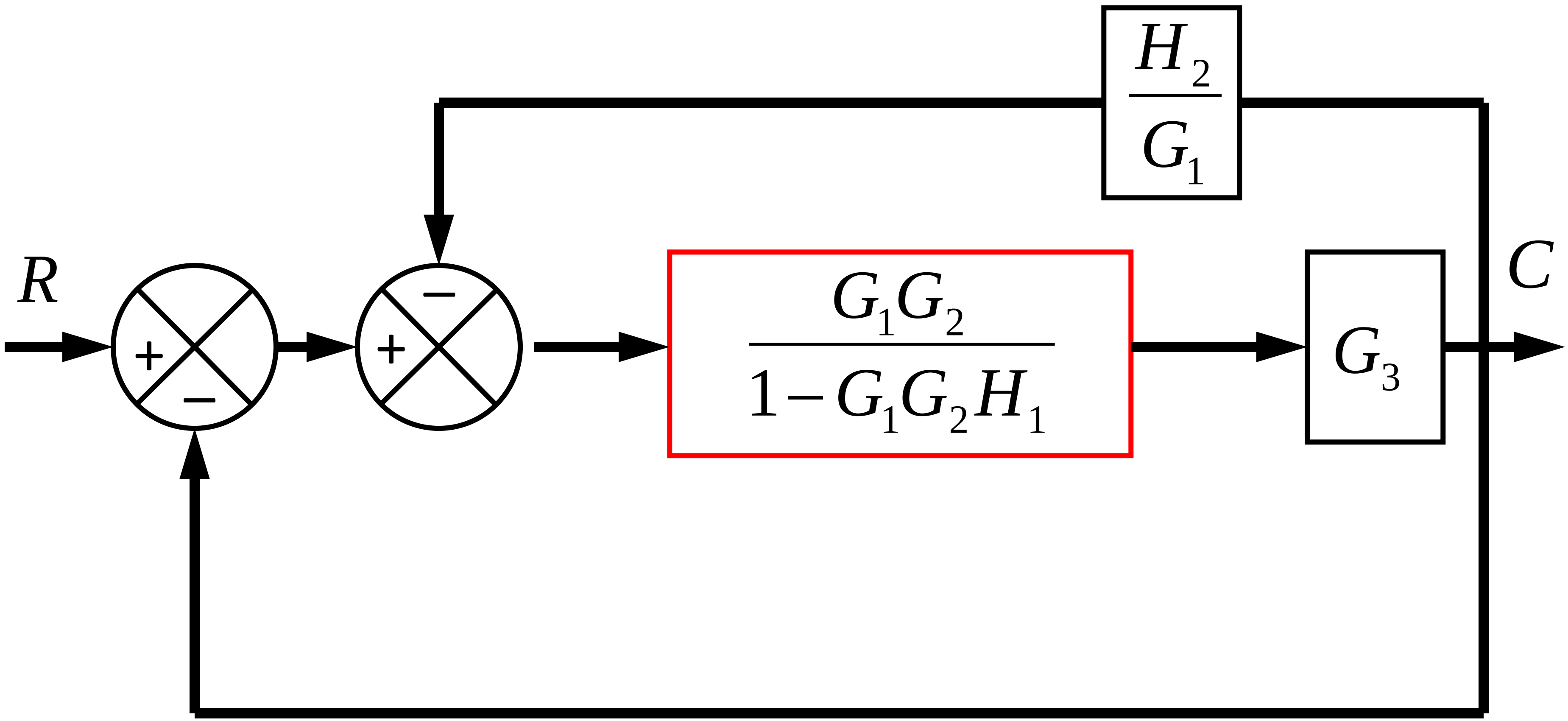
Example 10 : Cont.



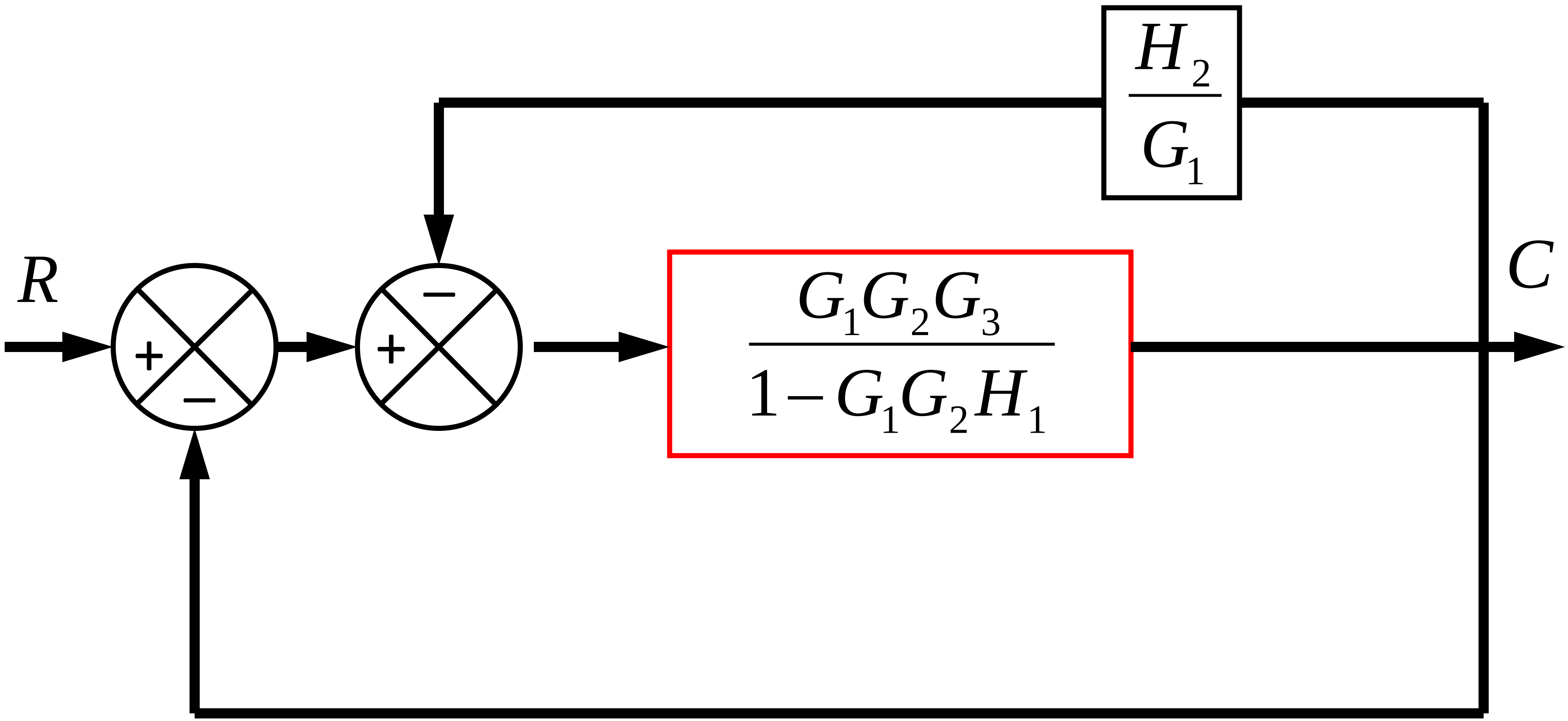
Example 10 : Cont.



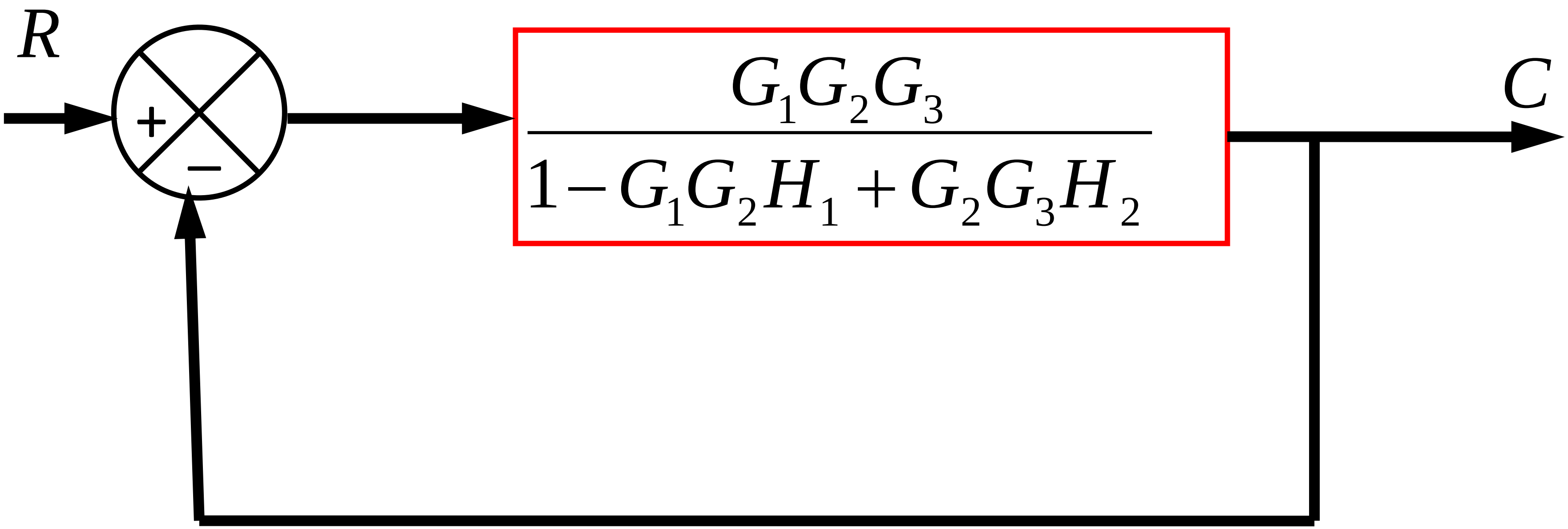
Example 10: Cont.

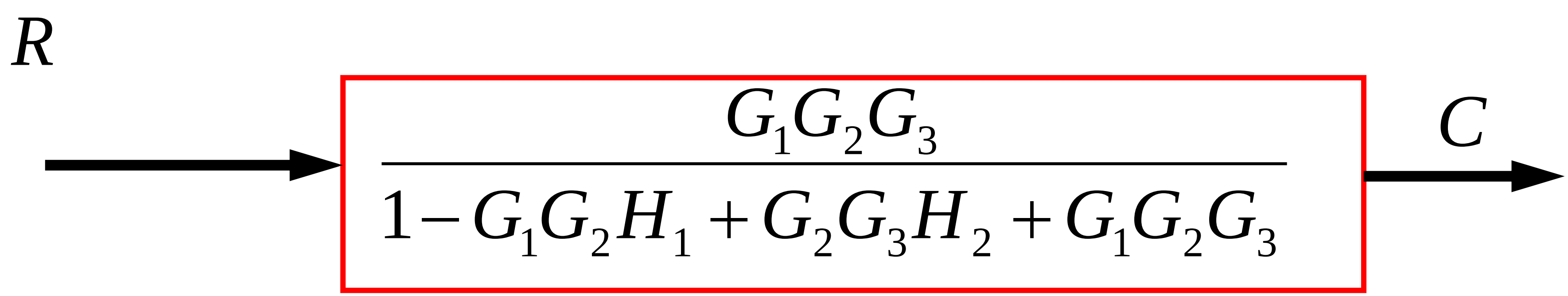


Example 10 : Cont.



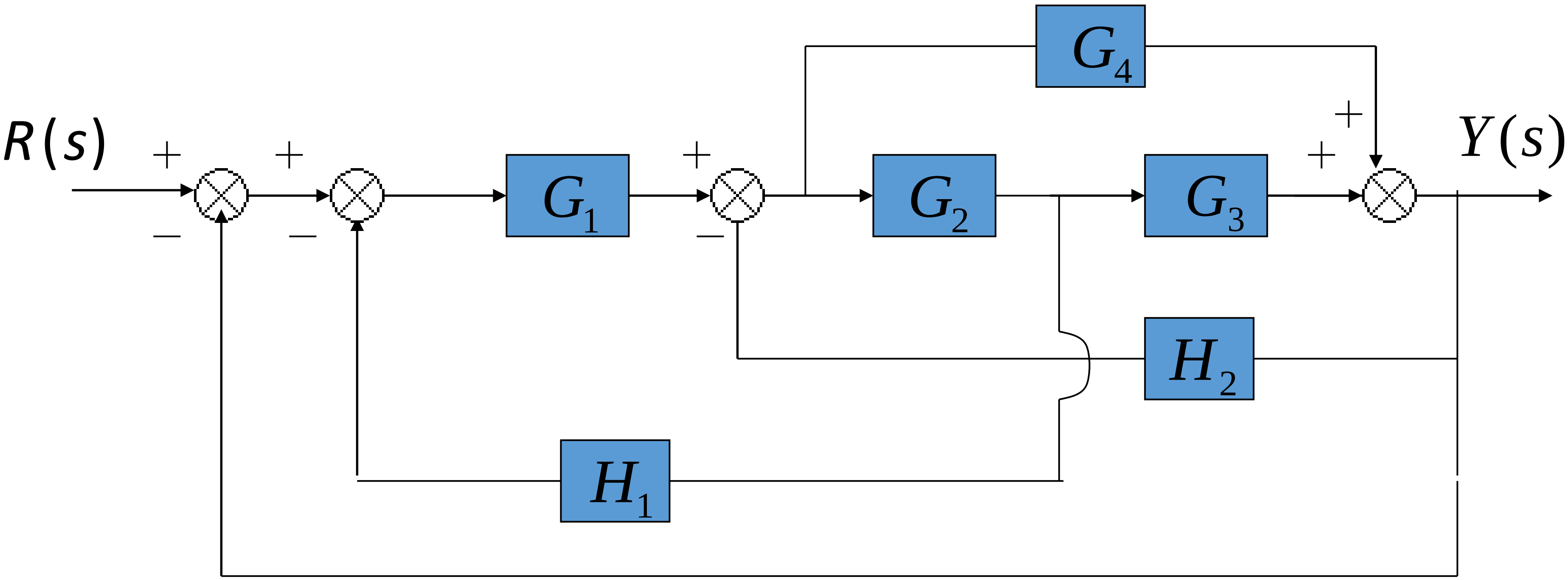
Example 10 (Cont.)

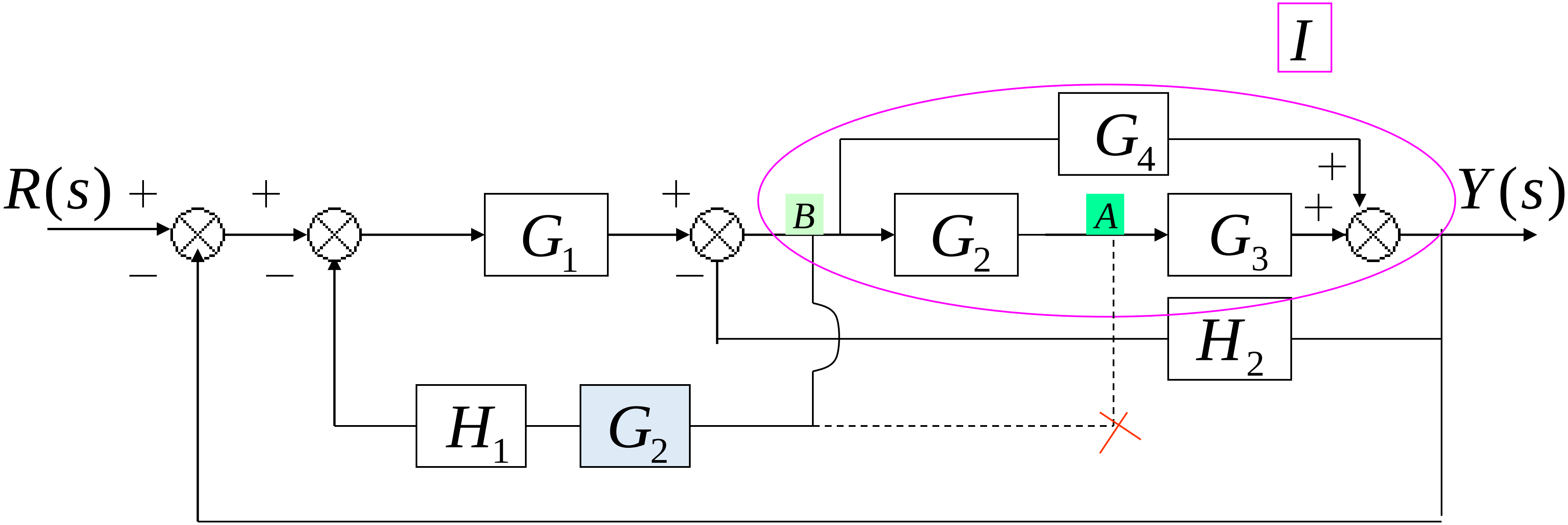


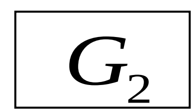


Example 11 :

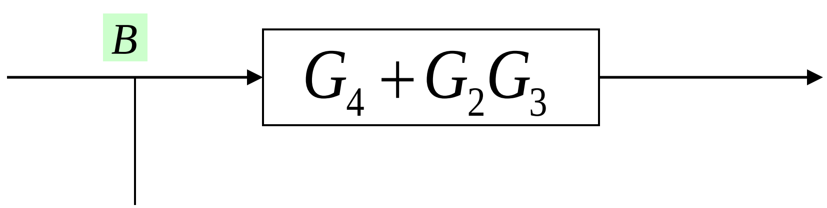
Find the transfer function of the following block diagram

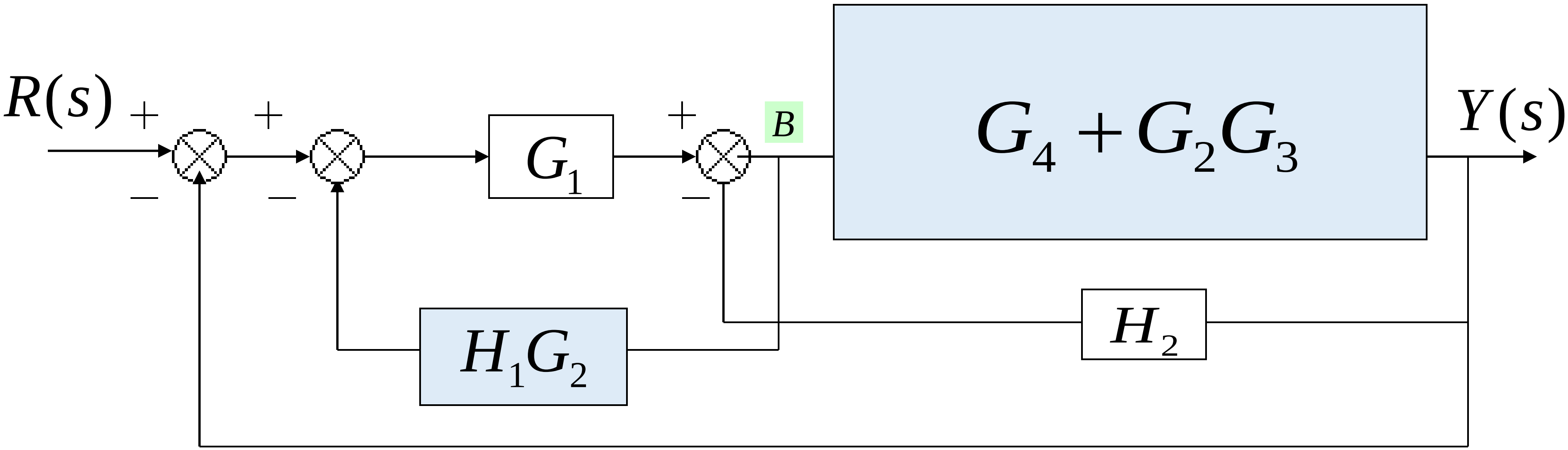




1. Moving pickoff point A ahead of block 

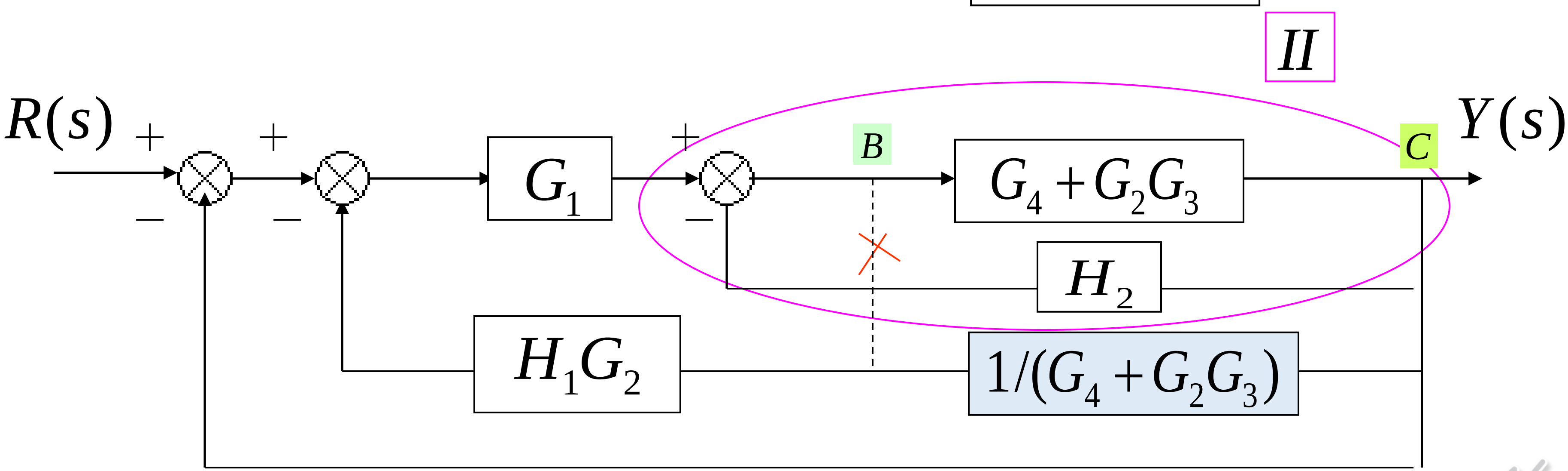
2. Eliminate loop I & simplify



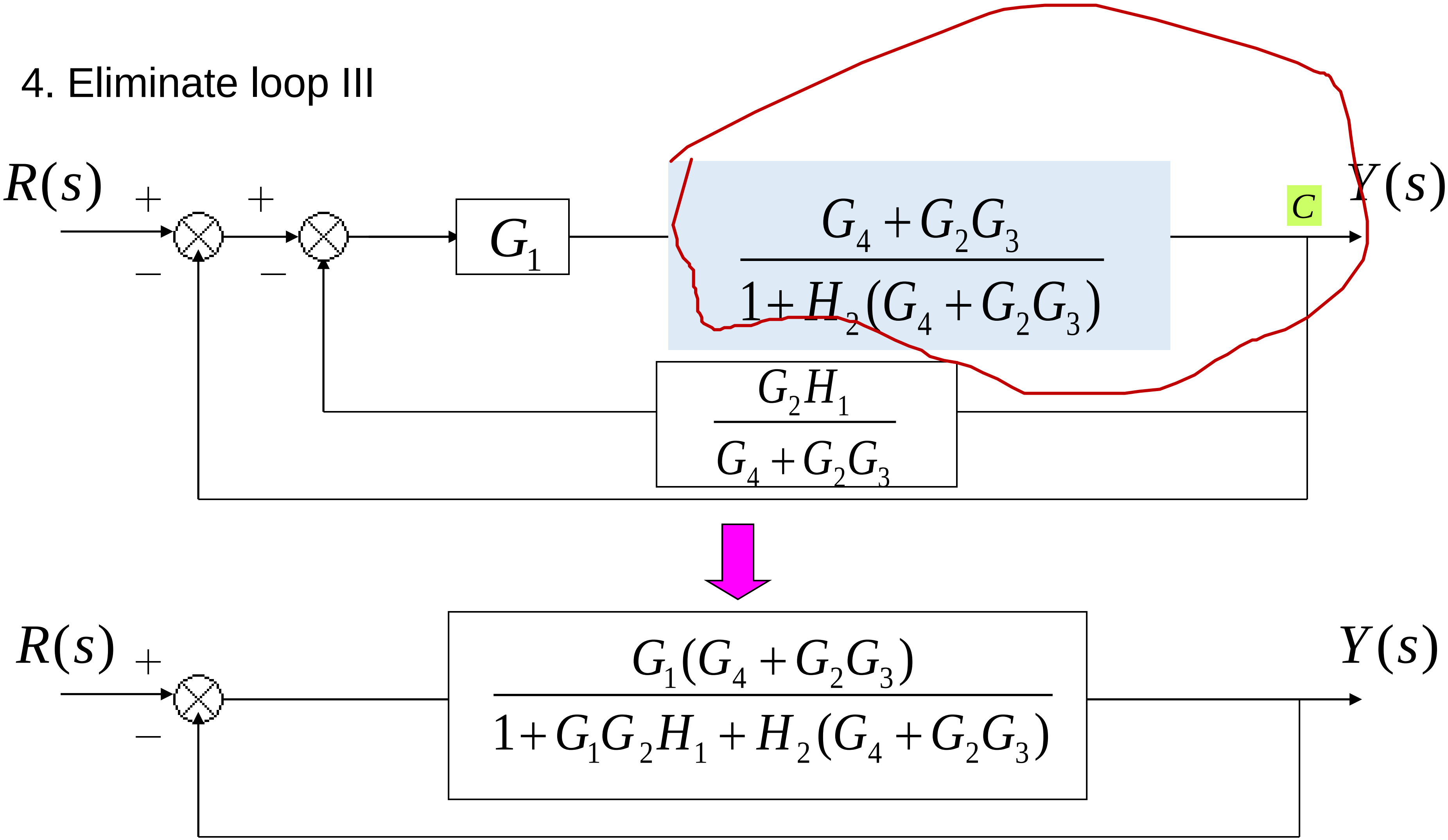


3. Moving pickoff point B behind block

$G_4 + G_2G_3$



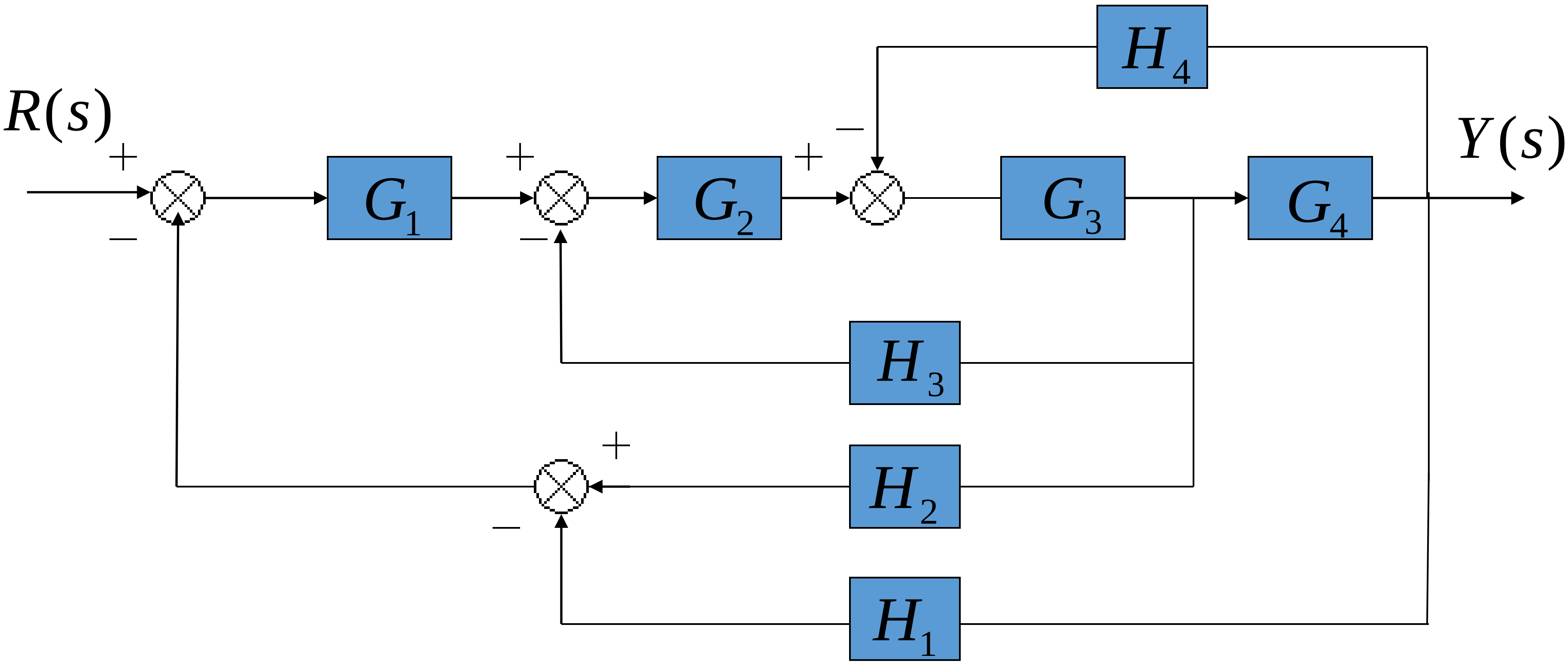
4. Eliminate loop III



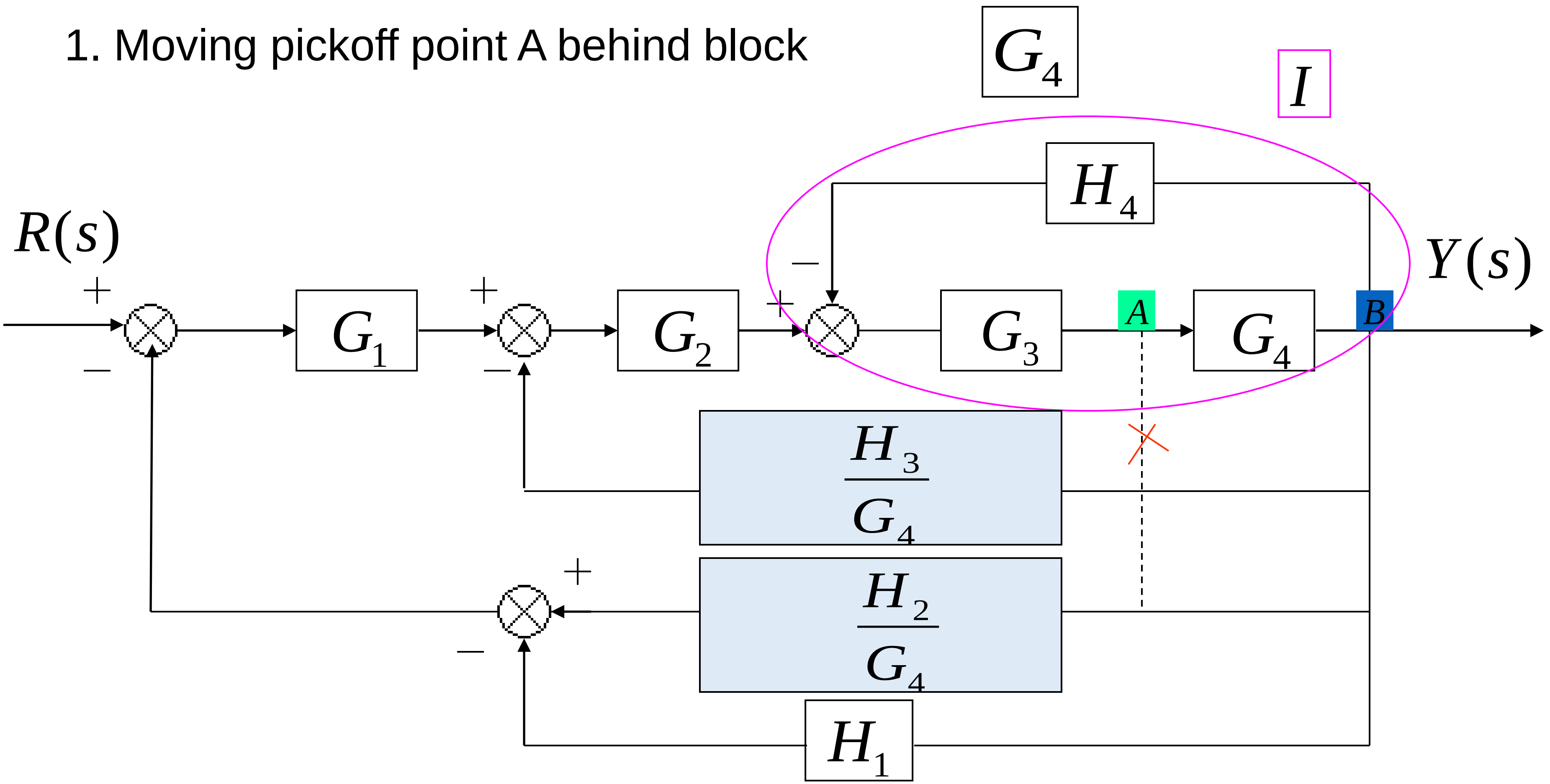
$$\frac{Y(s)}{R(s)} = \frac{G_1(G_4 + G_2G_3)}{1 + G_1G_2H_1 + H_2(G_4 + G_2G_3) + G_1(G_4 + G_2G_3)}$$

Example 12:

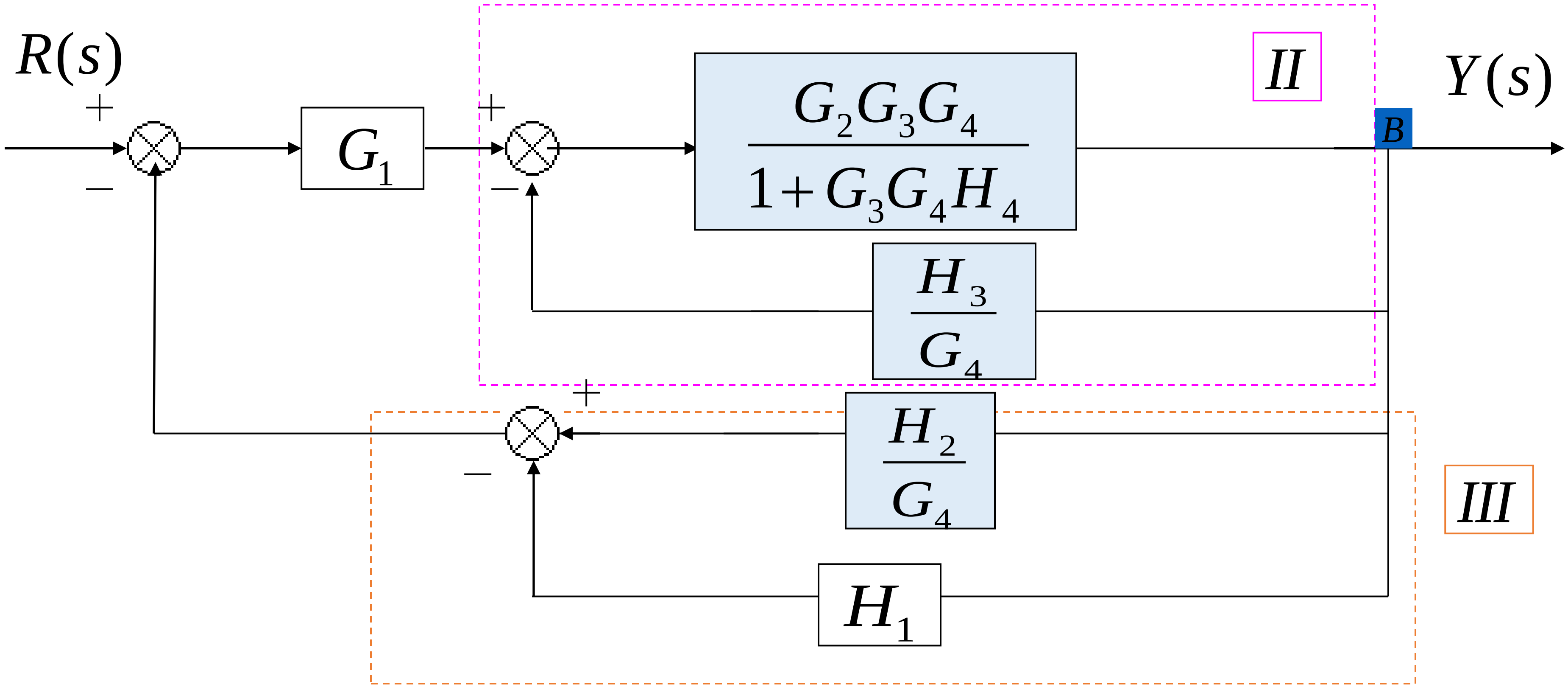
Find the transfer function of the following block diagrams



1. Moving pickoff point A behind block



2. Eliminate loop I and Simplify



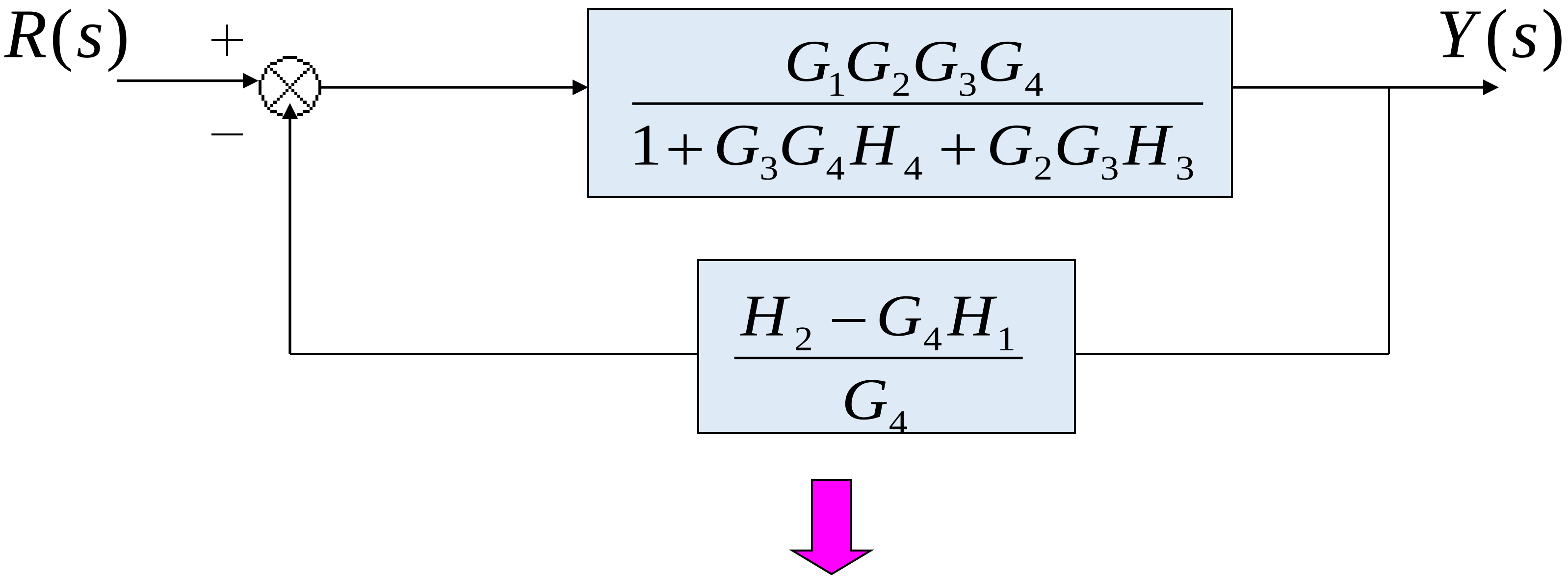
II feedback

$$\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III Not feedback

$$\frac{H_2-G_4H_1}{G_4}$$

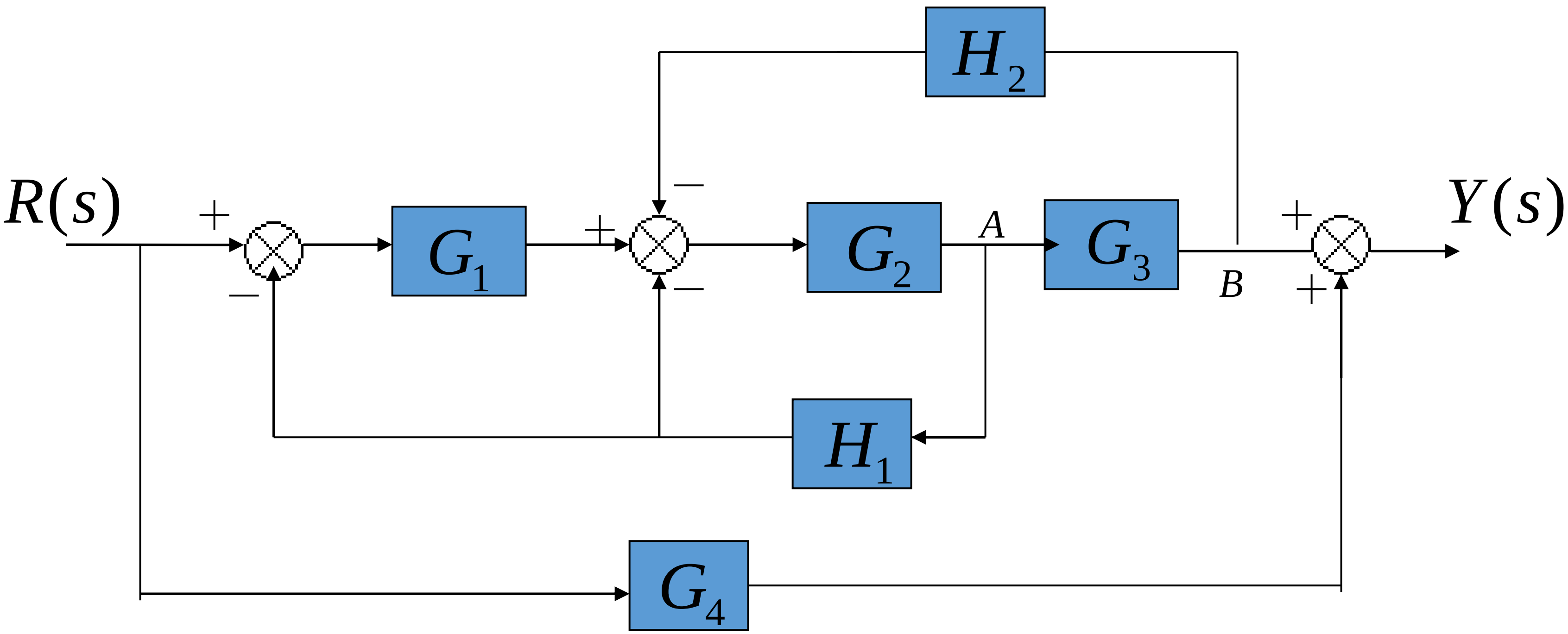
3. Eliminate loop II & IIII



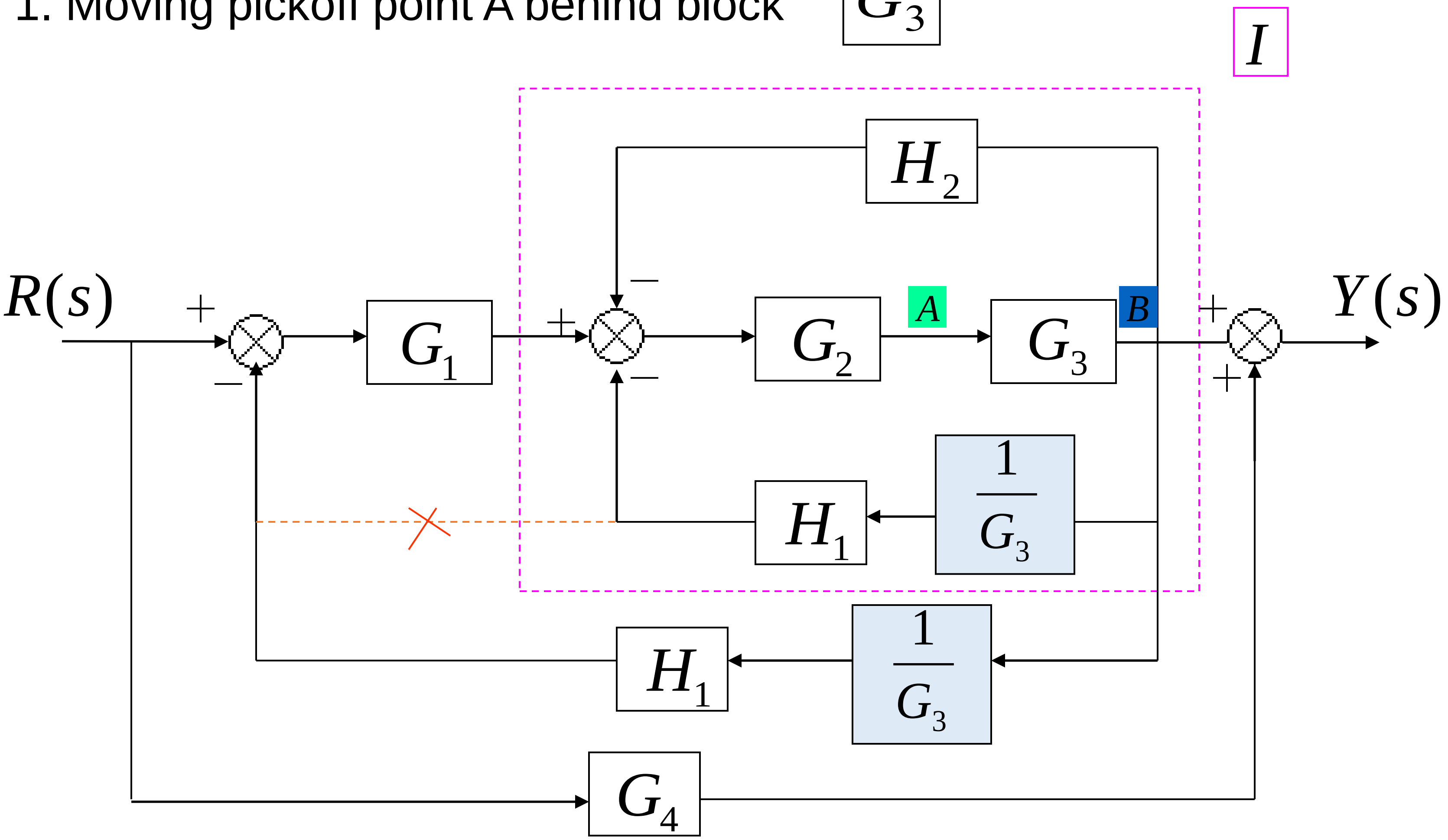
$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

Example 13:

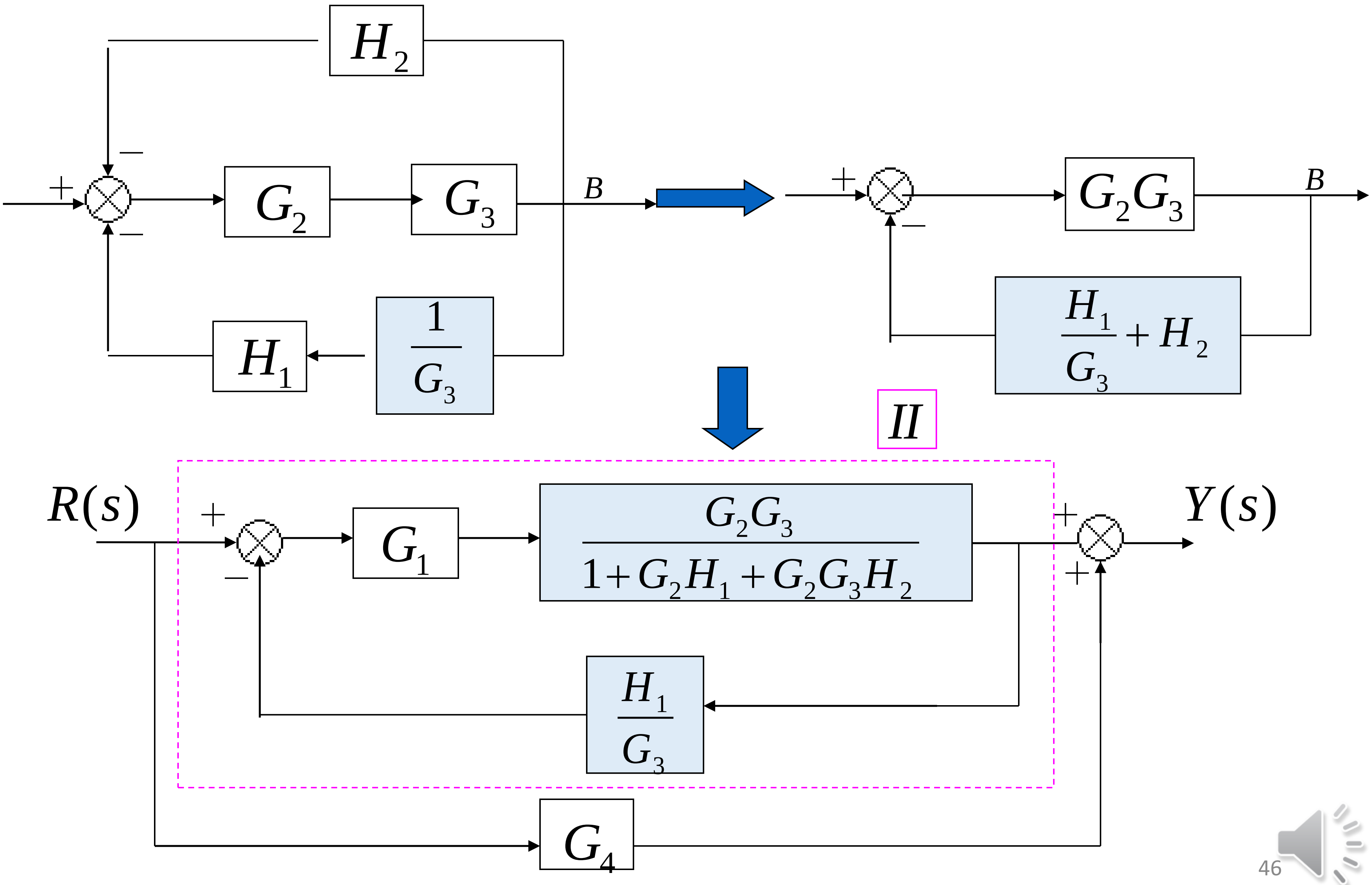
Find the transfer function of the following block diagrams



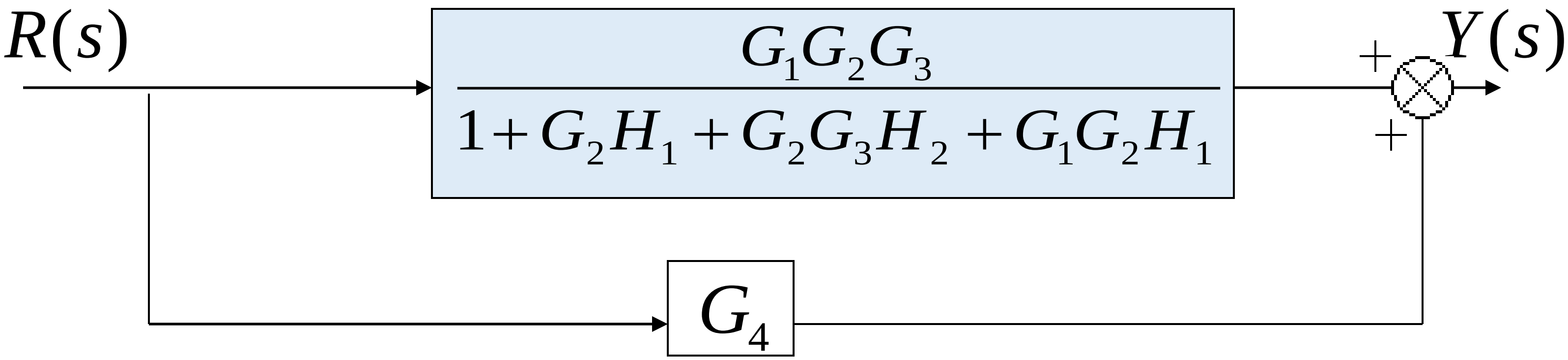
1. Moving pickoff point A behind block G_3



2. Eliminate loop I & Simplify

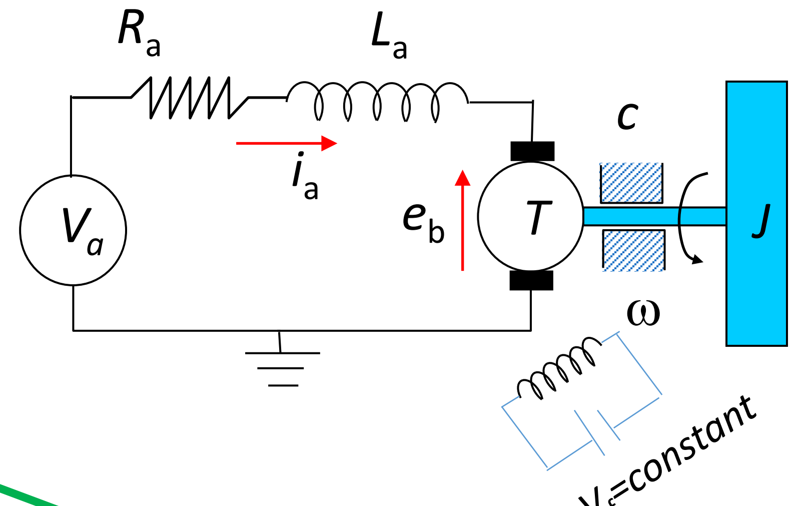


3. Eliminate loop II



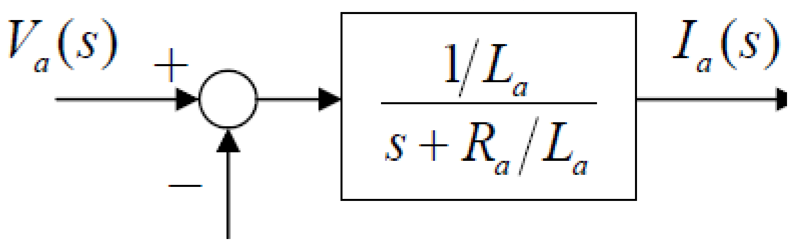
$$T(s)=\frac{Y(s)}{R(s)}=G_4+\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2+G_1G_2H_1}$$

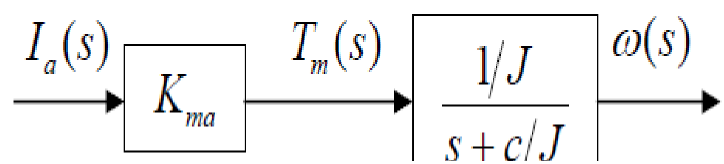
Example 14: Find Block Diagram of Armature Controlled D.C Motor

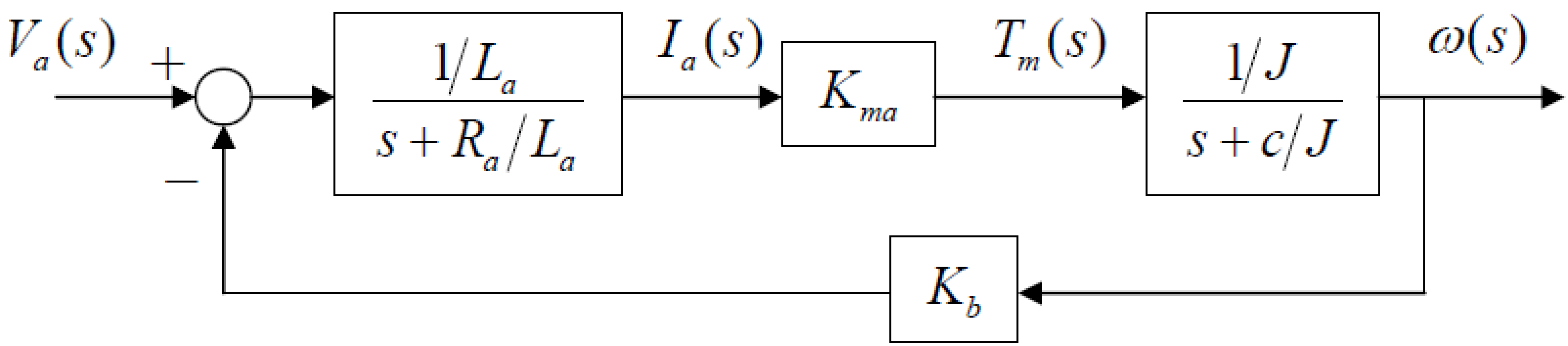


$$(L_a s + R_a)I_a(s) + K_b\omega(s) = V_a(s)$$

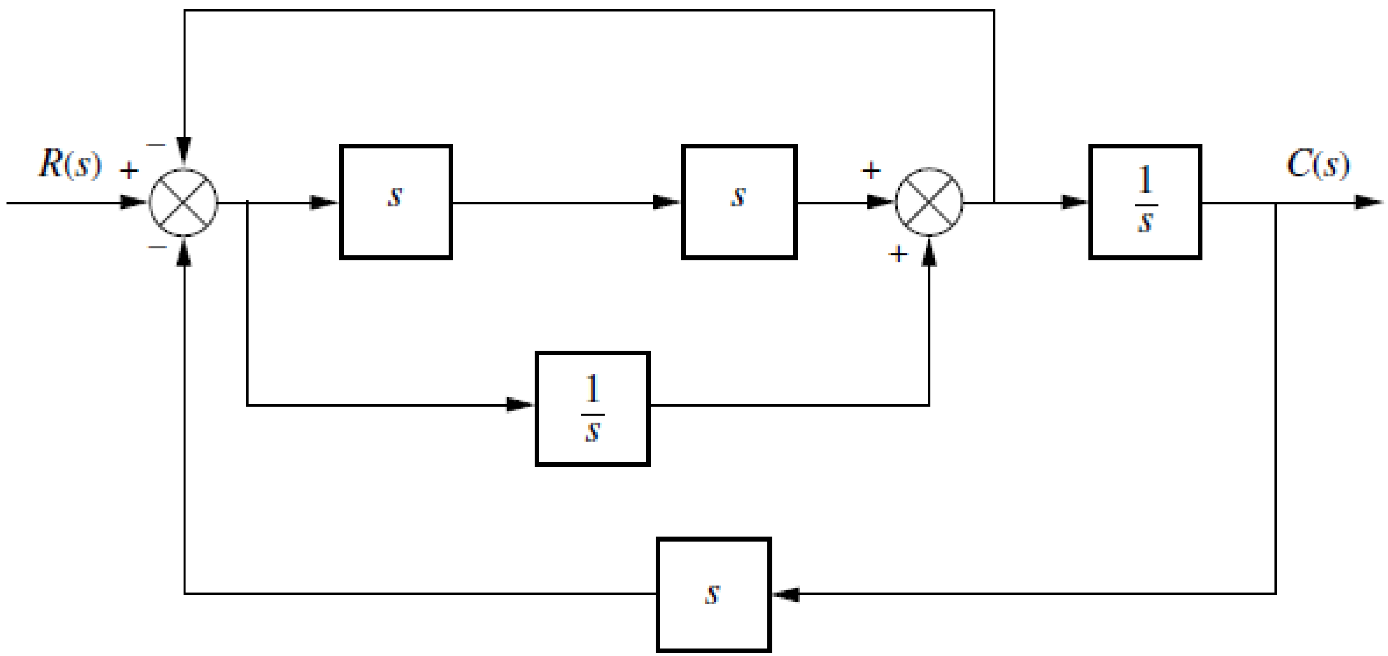
$$(Js + c)\omega(s) = K_{ma} I_a(s)$$







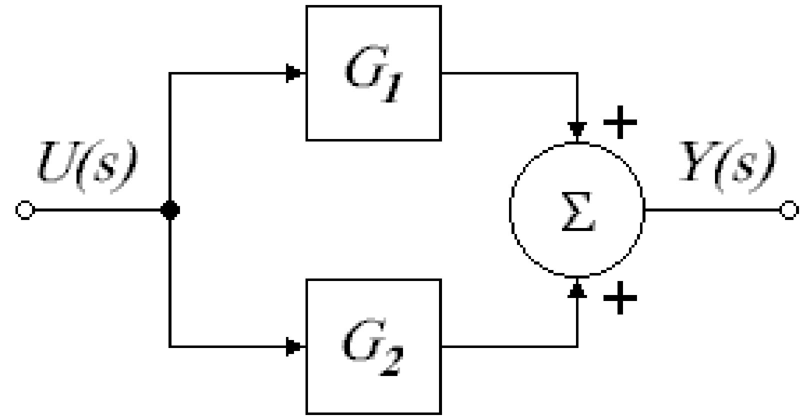
QUIZ: Find The TF



ANSWER: $$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

§ 5.4 Block Diagram Reduction using MATLAB

- MATLAB's Control Toolbox provides useful tools for manipulating block diagrams of linear systems.
- There are three basic configurations, the parallel, series, and feedback configurations.
- **(1) Parallel:**



For parallel configuration then the built in command "parallel" is used.

The syntax is

sys=parallel(sys1,sys2)

$$\frac{Y(s)}{U(s)} = G_1 + G_2$$

Example 15: $G_1(s) = \frac{s}{s^2+2}$, $G_2(s) = \frac{1}{3s+1}$

MATLAB code:

```
>>clc
>>clear all;
>>close all;
>>num1=[1 0];
>>denum1=[1 0 2];
>>sys1=tf(num1,denum1)
>>num2=[1];
>>denum2=[3 1];
>>sys2=tf(num2,denum2)
>>sys=parallel(sys1,sys2)
```

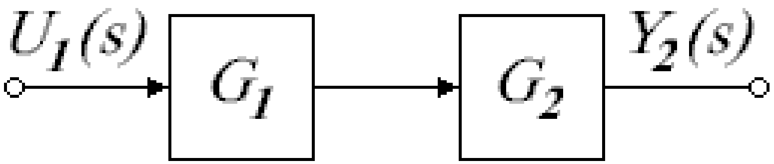
```
sys1 =
       s
   -------
   s^2 + 2
sys2 =
       1
   -------
   3 s + 1

sys =
      4 s^2 + s + 2
   ---------------------
   3 s^3 + s^2 + 6 s + 2
```

Block Diagram Reduction using MATLAB (Cont.)

- (**2) Series/Cascade Blocks:**

A series connection of transfer functions yields an overall transfer function of T(s) = G1(s) G2(s).

The MATLAB function series() can be used to determine this transfer function.



Assume two transfer functions, G1(s) =sys1, and G2(s) =sys2 the series connection is:

$$\frac{Y_2(s)}{U_1(s)} = G_2 G_1$$

sys=series(sys1,sys2)

Example 16: $G_1(s) = \frac{s}{s^2+2}$, $G_2(s) = \frac{1}{3s+1}$

```
>> clc
clear all;
close all;
num1=[1 0];
denum1=[1 0 2];
sys1=tf(num1,denum1)
num2=[1];
denum2=[3 1];
sys2=tf(num2,denum2)
sys=series(sys1,sys2)
```

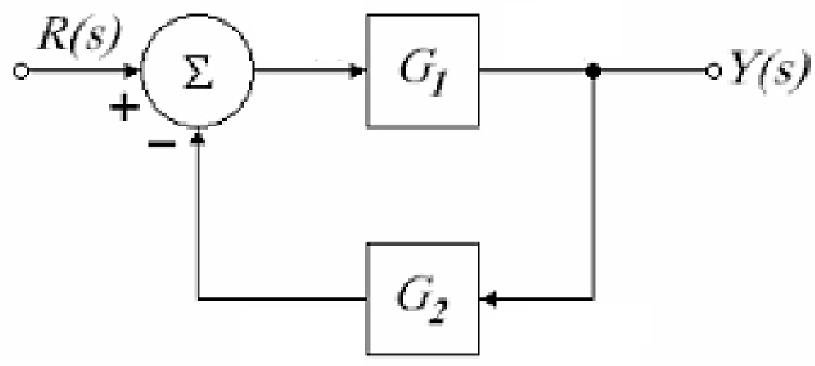
```
sys1 =
      s
   -------
   s^2 + 2
 sys2 =
      1
   -------
   3 s + 1

sys =
            s
   --------------------
   3 s^3 + s^2 + 6 s + 2
```

Block Diagram Reduction using MATLAB

- (3) **Feedback Blocks:**

For two blocks in a feedback loop, G1(s) and G2(s),

the overall transfer function,



$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 G_2}$$

The MATLAB function feedback

can be used to determine the

overall transfer function of this system.

sys=feedback(sys1,sys2)

Example 17: $G_1(s) = \frac{s}{s^2+2}$, $G_2(s) = \frac{1}{3s+1}$

```
>> clc
clear all;
close all;
num1=[1 0];
denum1=[1 0 2];
sys1=tf(num1,denum1)
num2=[1];
denum2=[3 1];
sys2=tf(num2,denum2)
sys=feedback(sys1,sys2)
```

```
sys1 =
      s
  -------
  s^2 + 2

sys2 =
      1
  -------
  3 s + 1

sys =

      3 s^2 + s
  ---------------------
  3 s^3 + s^2 + 7 s + 2
```

Thank You